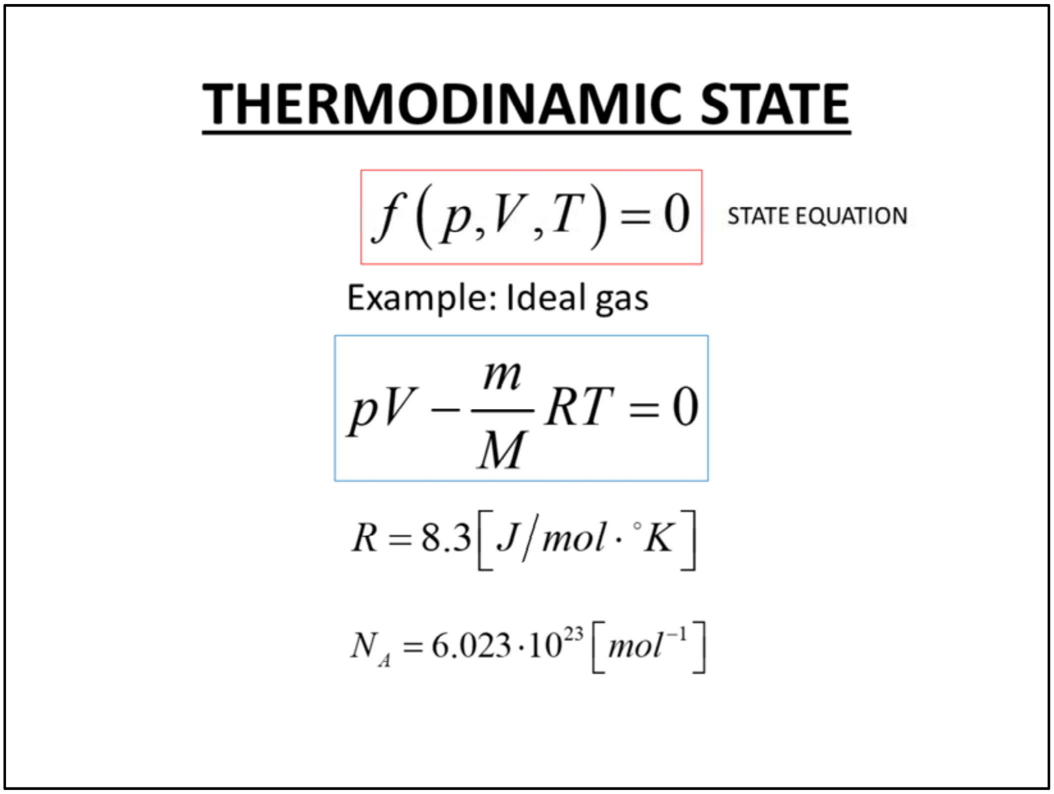
OPTICAL TRANSMITTERS



THERMODINAMIC STATE

$$\boxed{f\left(p,V,T\right)=0}$$

STATE EQUATION

Example: Ideal gas

$$\boxed{pV-\frac{m}{M}RT=0}$$

$$R=8.3\left[J/mol\cdot{}^{\circ}K\right]$$

$$N_A=6.023\cdot10^{23}\left[mol^{-1}\right]$$

The state of a mechanic system is fully described at a given point in time by the position and speed of each material point of the system. For a system composed of N material points, it is therefore necessary to know 6N variables.

Thermodynamic systems contain a very large number of material points (atoms or molecules), making it impossible to describe them using the 6N variables. For this reason, another system state concept needs to be defined.
For a system that contains a certain quantity of substance m, the temperature T, volume V and pressure p are the physical quantities that describe the thermodynamic state of the system. They are not independent; they are linked by the equation of state. The thermodynamic state of a system is therefore completely determined by two of the three quantities, p, V, T.

If, for example, we represent (V, p) in a rectangular coordinate system, a point in the plane (V, p) represents a thermodynamic state of the system. If we fix it on T, the points corresponding to states of equal temperature are on a curve called isothermal.
Knowing the thermodynamic state of a system is NOT sufficient to determine its mechanical state. A given thermodynamic state (e.g., temperature and volume) corresponds to an infinity of mechanical states of the molecules. For this reason, we will want to consider the average mechanical behavior calculated on this infinite set of mechanical states. Statistical mechanics deals with this problem.

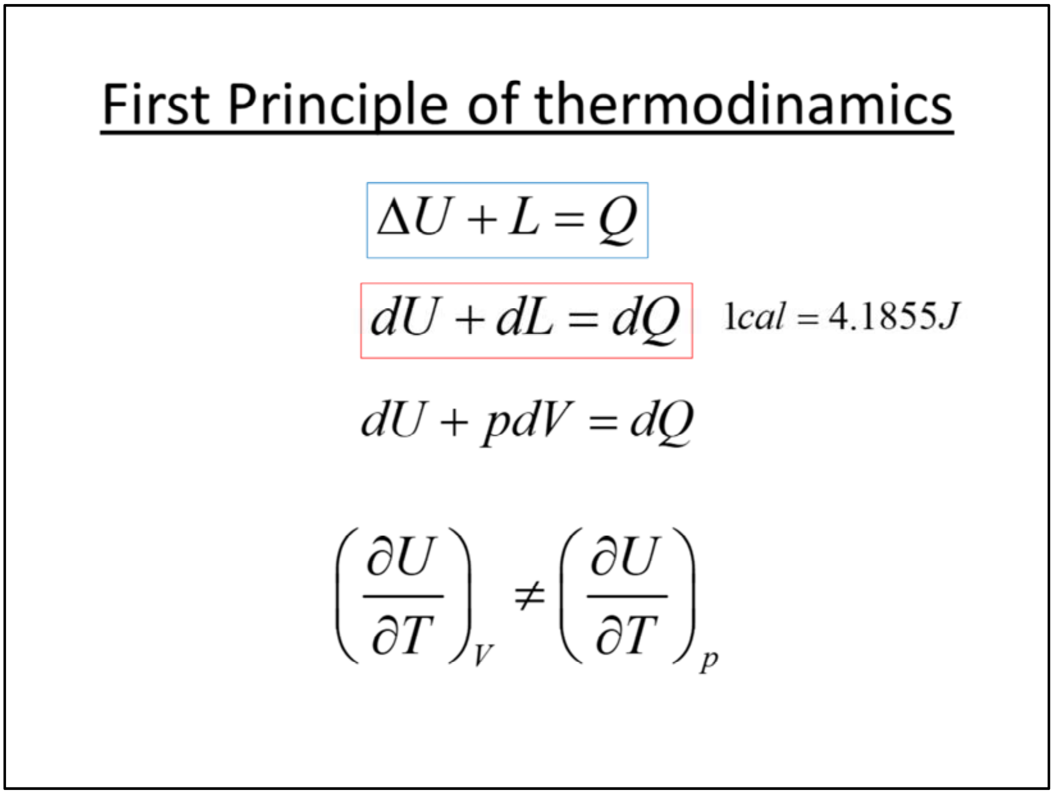
Among the thermodynamic states of a system, of particular importance are the states of equilibrium. Steady state is the state that does not change as long as external conditions remain unchanged.

The transformation of a system is the transition from time to time, from an initial state to a final state through a continuous succession of intermediate states.

Reversible transformation is that transformation in which successive states differ infinitesimal by steady states. Therefore, the initial state and the final state of a reversible transformation must still be steady state.

Cyclical transformation is the transformation in which the initial state and the final state are the same.

Example: Ideal gas. R = universal gas consumption = 8.3 J / (mol * K)



First Principle of thermodinamics

$$\boxed{\Delta U + L = Q}$$

$$\boxed{dU + dL = dQ} \quad 1cal = 4.1855J$$

$$dU + pdV = dQ$$

$$\left(\frac{\partial U}{\partial T}\right)_V \neq \left(\frac{\partial U}{\partial T}\right)_p$$

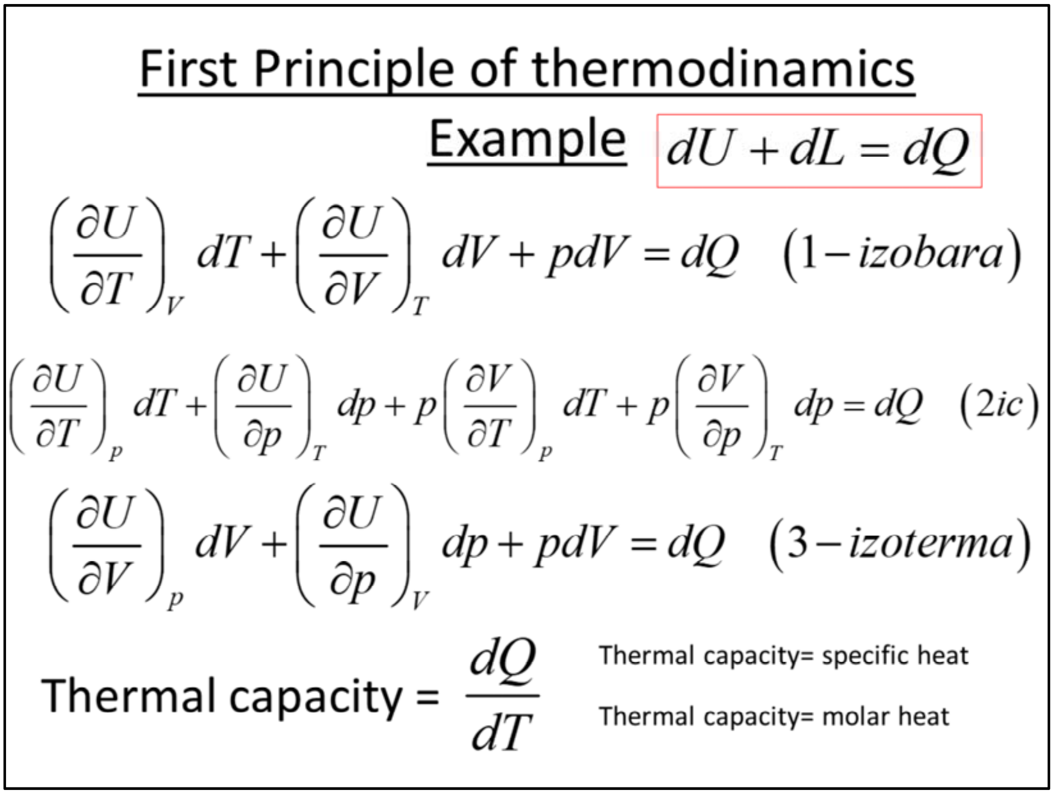
Principle 1 of thermodynamics is the principle of energy conservation for thermodynamic systems: the system energy variation during a transformation must be equal to the energy that the system has received from the environment.

What does "system energy" mean and "energy received from the environment"?

Heat is any form of energy, other than mechanical, that is supplied to the outside system.

Heat is measured in calories. 1 cal = 4.1855 J.

The energy of the system, U, depends only on the state of the system and therefore it is a function of the two sizes chosen to represent the state of the system.



First Principle of thermodinamics

Example $dU + dL = dQ$

$$\left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV + pdV = dQ \quad (1-izobara)$$

$$\left(\frac{\partial U}{\partial T}\right)_p dT + \left(\frac{\partial U}{\partial p}\right)_T dp + p\left(\frac{\partial V}{\partial T}\right)_p dT + p\left(\frac{\partial V}{\partial p}\right)_T dp = dQ \quad (2ic)$$

$$\left(\frac{\partial U}{\partial V}\right)_p dV + \left(\frac{\partial U}{\partial p}\right)_V dp + pdV = dQ \quad (3-izoterma)$$

Thermal capacity = $\dfrac{dQ}{dT}$

Thermal capacity= specific heat

Thermal capacity= molar heat

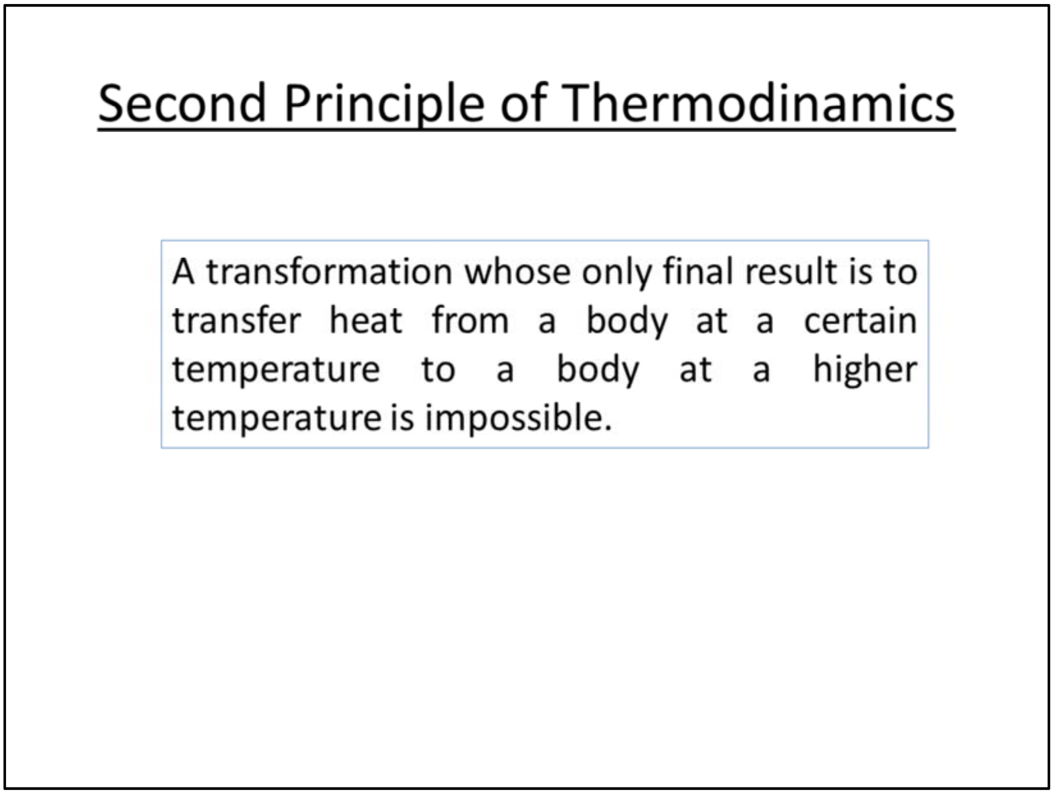
Example

We consider an infinitesimal transformation of the system, a transformation in which the independent variables vary in infinitesimal quantities. We consider that the state of the system is represented on a diagram (V, p).
For T and V independent variables we have (1)
For T and p independent variables we have (2)
For V and p independent variables we have (3)

Thermal capacity: the ratio of the infinitesimal quantity of heat, dQ, absorbed by a body and the infinitesimal temperature increase, dT, produced by this heat.

If the body has 1g, then thermal capacity is called specific heat.

If the body has a substance amount of 1mol, then the thermal capacity is called molar heat.



Second Principle of Thermodinamics

A transformation whose only final result is to transfer heat from a body at a certain temperature to a body at a higher temperature is impossible.

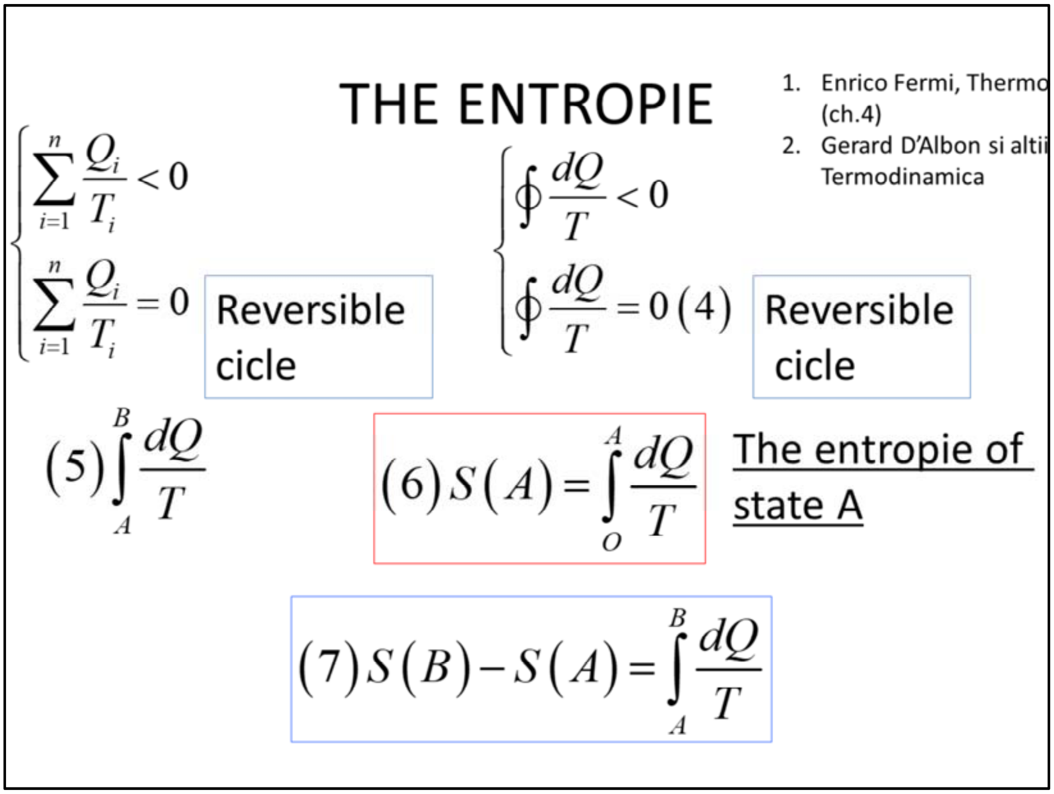
First Principle establishes the impossibility of building a machine that can create energy. But it does not introduce any limitation in the possibility of transforming one energy from one form into another.

This is certainly true for transforming an energy into heat. A body at a certain temperature can always be heated by friction. Electricity can always be transformed into heat by passing an electric current through a resistance.

There are, however, very limited cases of heat transformation in mechanical work.

We call <u>heat source at temperature T</u>, a body that has the temperature T in every point and which it is conditioned in such a way that it can only change heat and not the mechanical work with the environment.

<u>A transformation whose only final result is to transfer heat from a body at a certain temperature to a body at a higher temperature is impossible.</u>



THE ENTROPIE

1. Enrico Fermi, Thermo (ch.4)
2. Gerard D'Albon si altii Termodinamica

$$\begin{cases} \sum_{i=1}^{n} \frac{Q_i}{T_i} < 0 \\ \sum_{i=1}^{n} \frac{Q_i}{T_i} = 0 \end{cases}$$

Reversible cicle

$$\begin{cases} \oint \frac{dQ}{T} < 0 \\ \oint \frac{dQ}{T} = 0 \; (4) \end{cases}$$

Reversible cicle

$$(5) \int_{A}^{B} \frac{dQ}{T}$$

$$\boxed{(6)\, S(A) = \int_{O}^{A} \frac{dQ}{T}}$$

<u>The entropie of state A</u>

$$\boxed{(7)\, S(B) - S(A) = \int_{A}^{B} \frac{dQ}{T}}$$

We consider a system S that undergoes a cyclical transformation.
Suppose that during the cycle, the system receives heat from a lot of sources at temperatures T1, T2, ..., Tn. Let the quantities of heat exchanged between the system and these sources to be Q1, Q2, ..., Qn. Q will be considered positive if it is the heat received by the system and negative otherwise.

If the system changes the heat with a continuous distribution of sources, the sum must be replaced by an integral over the entire cycle. dQ beeing an infinitezimal amount of heat received by the system from a source at temperature T, we have the relationship (4).

In the case of a reversible cycle, the relation (4) can also be formulated as follows. Let A and B be two equilibrium states of the system S. Consider a reversible transformation that brings the system from the initial state A to the final state B. Integrala (5) extends over the reversible transformation from A to B (dQ being the amount of heat received reversibly by the system at T temperature). This integral has the same value for any reversible transformation from A to B; so its value depends only on extreme states A and B and does not depend on the reversible transformation that links these states.

This property allows us to define a new function that depends only on the state of the system: <u>entropy</u>. If we arbitrarily choose a state of equilibrium of the system, O, that we call <u>the standard state</u>, then A beeing an another state of equilibrium, its entropy will be given by the relation (6) in which the integral is taken over a reversible transformation. We already know that this integral depends only on the states O and A and not on the reversible transformation chosen. Since the standard state O is fixed, we can say that the function S (A) depends only on the state A, and it is named <u>the entropy of state A</u>.
Entropy is defined until an arbitrary constant. Principle 3 of thermodynamics completes the definition of entropy and allows us to determine the entropy constant.

Entropie's Properties

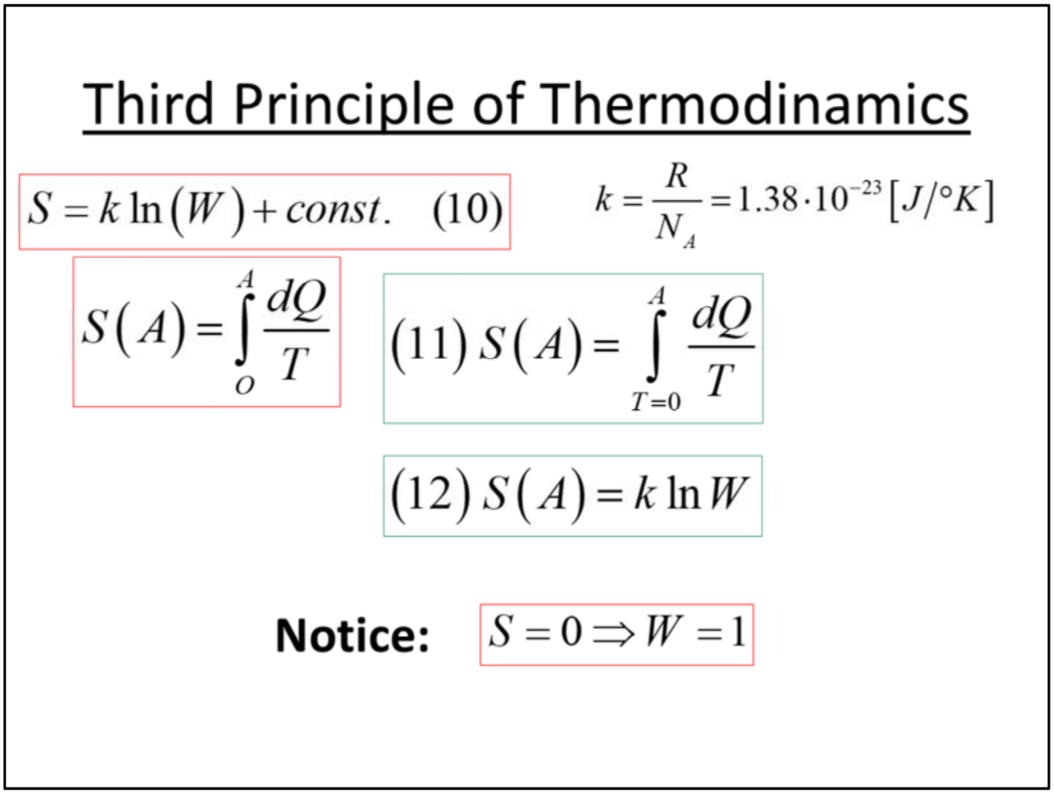
$$(8)\begin{cases} S(B)-S(A)=\int_A^B \frac{dQ}{T} & \text{Reversible Transformation} \\ S(B)-S(A)>\int_A^B \frac{dQ}{T} & \text{Ireversible Transformation} \end{cases}$$

$$(9)\begin{cases} dQ=0 & \text{Isolated System} \\ S(B)>S(A) & \text{Ireversible transformation in an isolated system} \\ S(B)=S(A) & \text{Reversible transformation in an isolated system} \end{cases}$$

In any transformation that occurs in an isolated system, the entropy increases if transformation is irreversible. In the case of reversible transformation, entropy does NOT change during transformation.

To reduce the entropy of a system, we need to open it and put it in contact with a second system. The total entropy of the two systems can not decrease.

When an isolated system is in the state of maximum entropy, it can no longer undergo any transformation because any transformation should lead to an increase in entropy. So the state of maximum entropy is the most stable state of the isolated system.



Third Principle of Thermodinamics

$$S = k \ln(W) + const. \quad (10)$$

$$k = \frac{R}{N_A} = 1.38 \cdot 10^{-23} [J/°K]$$

$$S(A) = \int_O^A \frac{dQ}{T}$$

$$(11)\, S(A) = \int_{T=0}^{A} \frac{dQ}{T}$$

$$(12)\, S(A) = k \ln W$$

Notice: $S = 0 \Rightarrow W = 1$

Boltzmann has shown that the entropy of a state of a thermodynamic system is linked by a simple relationship to the thermodynamic probability, W, of that state, eq.(10).
We have already evoked the difference between the notions of dynamic state and of thermodynamic state of a system. The same thermodynamic states correspond to a very large number of dynamic states. In statistical mechanics are given the criteria for assigning to a given thermodynamic state, a W number of the dynamic states corresponding to them. This number is usulay called the probability of the given thermodynamic state, but, strictly speaking, it is only proportional to the probability defined in the usual sense. The latter can be obtained by dividing W at the total number of possible dynamic states.
According to the statistical considerations, in an isolated system, only those spontaneous transformations can occur that lead the system into higher probability states, so that the most stable state of the system will be the state with the highest probability and which is in accordance with the total energy of the system.

In the definition of entropy, there is an initial state, O, arbitrarily chosen, which is in accordance with the arbitrary constant of the Boltzmann relation.

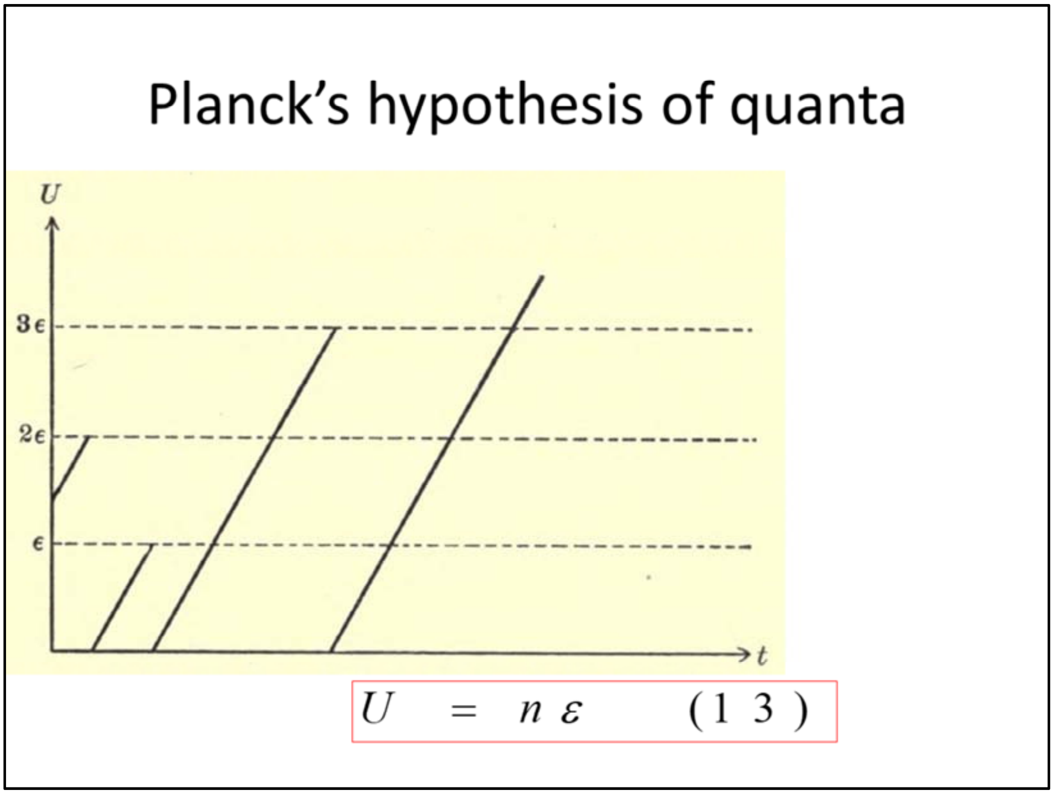
Principle 3 of thermodynamics (also known as Nernst's theorem) allows the determination of the additive constant.

Nernst's Theorem: The entropy of any system at temperature of absolute zero can always be considered zero.

Nernst's theorem has to be interpreted physically as that all possible states of the system, at T = 0, have to have the same entropy.

Therefore, it is obvious that we will choose as the standard state of the system, the states at T = 0. This will allow us to fix the entropy of the standard state as zero, eq. (11). In relation (11), the integral is considered along a reversible transformation between any state of the system, at T = 0, and the state A. The constant in the Boltzmann's relation is therefore zero, eq.(12).

Because the value of W is 1 when entropy is zero, we can interpret, in statistical terms, that the thermodynamic state of a system at absolute zero corresponds to only one dynamic state, which is the dynamic state of lowest energy, compatible with the given crystalline structure or the aggregation state of the system.



Planck’s hypothesis of quanta

$$U = n\varepsilon \qquad (13)$$

An oscillator which absorbs energy must emits also, because only in such condition a stationary state would be possible.

Planck assumes that the emission does not occur continuously, but it takes place only at certain defined moments oftime, instantaneously, in impulse, and that the energy emission appears only when its vibrational energy, U, is a whole multiplicity of quantum energy epsilon. That the oscillator emits energy or it continues to increase his vibrational energy by absorption is a problem of chance. If the emission still occurs, however, the whole vibration energy U is emitted so that oscillator's vibration decreases to zero and then rises again with a new radiant energy absorption.

Thermodinamic probability

$$\boxed{S(A) = k \ln W \quad (14)}$$

$$U_N = NU \quad (15) \qquad\qquad S_N = NS \quad (16)$$

$$U_N = NU = Nn\varepsilon = P\varepsilon \quad (17)$$

$$S_N = k \ln W \quad (18)$$

Determining the thermodynamic probability of a system in a certain thermodynamic state requires more complete discussion of the notion of "state" of a physical system.
By the state of a physical system, at a certain point in time, we understand the aggregation of all mutually independent quantities, which uniquely determines the way the processes occur in the system over time under the given frontier conditions. Thus, knowing a state is equivalent to knowing the initial conditions. We need to distinguish between two completely different types of states: macroscopic states and microscopic states.

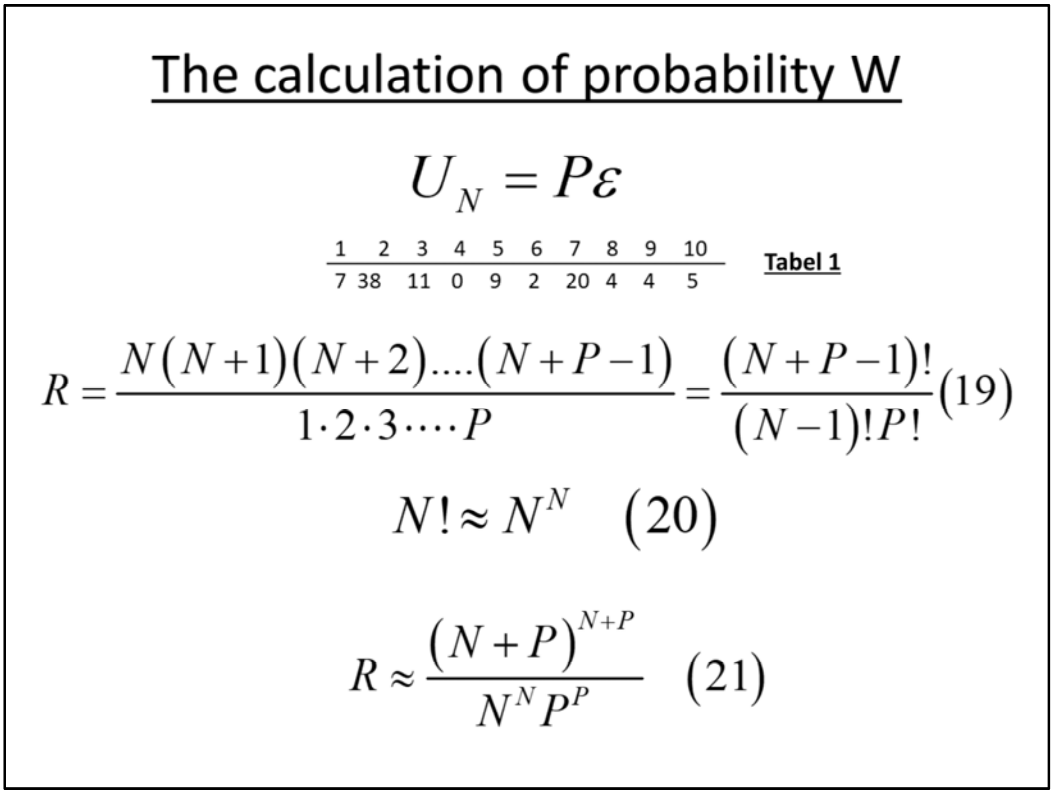
The microscopic state is the state described by a mechanical or electromagnetic observer; it contains separate values of all coordinates, velocities and field intensities. Microscopic processes, in accordance with mechanical and electromagnetic laws, take place in a perfectly deterministic form; for them, entropy and the second principle of thermodynamics have no meaning.

The macroscopic state is the one observed by a thermodynamic observer. Any macroscopic state contains a very large number of microscopic states, which the observer joins them in an average value. Thermodynamic processes take place in a non-ambiguous way from the point of view of the second principle of thermodynamics, and only when the hypothesis of elementary chaos is satisfied.

The relationship (14) contains a general method of calculating entropy S by probability considerations. This has no practical relevance unless the thermodynamic probability W of the system in a given state can be calculated.

Plank made this calculation by introducing the notion of quanta of energy.

The energy U of a resonator must be considered as the mean over time, which is the same as the simultaneous average of the energies of a very large number N of identical resonators located in the same radiation field and which are sufficiently separate to consider that they do not influence each other directly. In this sense, the total energy, UN, of the N resonator system will be (15), which corresponds to a total SN entropy given by the relation (16), where S represents the mean entropy of a single resonator. Entropy SN depends on the disorder in which total UN energy is distributed among individual resonators. So we have relationships (17) and (18).



The calculation of probability W

$$U_N = P\varepsilon$$

1	2	3	4	5	6	7	8	9	10
7	38	11	0	9	2	20	4	4	5

Tabel 1

$$R = \frac{N(N+1)(N+2)....(N+P-1)}{1\cdot 2\cdot 3\cdots\cdot P} = \frac{(N+P-1)!}{(N-1)!P!} \quad (19)$$

$$N! \approx N^N \quad (20)$$

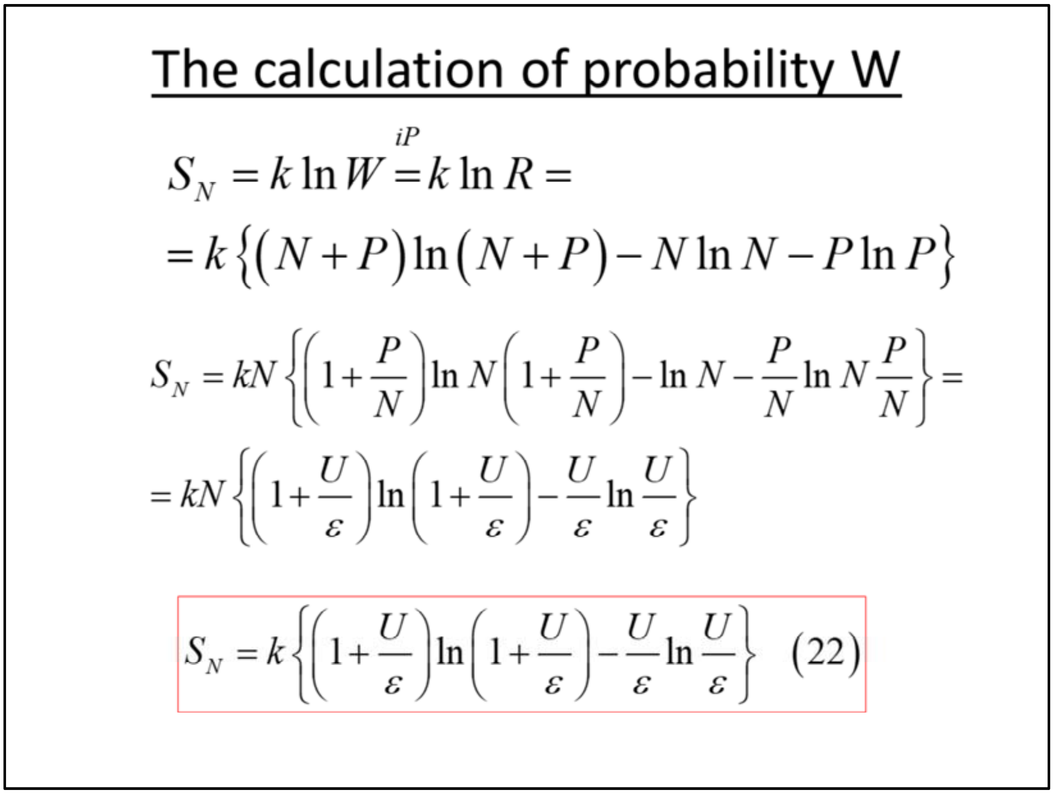
$$R \approx \frac{(N+P)^{N+P}}{N^N P^P} \quad (21)$$

The distribution of the P energy elements between the N resonators is done in a finite, defined and integer number. Each form of distribution is called (after Boltzmann) a "complex".
If we denote the resonators with 1, 2, 3, ... N, and write under each resonator the number of energy elements (quanta) allocated to it in a given random distribution, we obtain for each complex a configuration as in Table 1. In the example we assumed N = 10 and P = 100.

The number R of possible complexes is equal to the number of arrangements we can obtain in this way for the bottom line for an N and P given. For clarity, we specify that two complexes must be considered different if the corresponding number's configurations contains the same numbers, but in diffrent orders.

From the combinatorics, we get the number of possible complexes (19)

According to Stirling's theorem, we have the approximate (20) by which (19) becomes (21).



The calculation of probability W

$$S_N = k\ln W \overset{iP}{=} k\ln R =$$

$$= k\left\{(N+P)\ln(N+P) - N\ln N - P\ln P\right\}$$

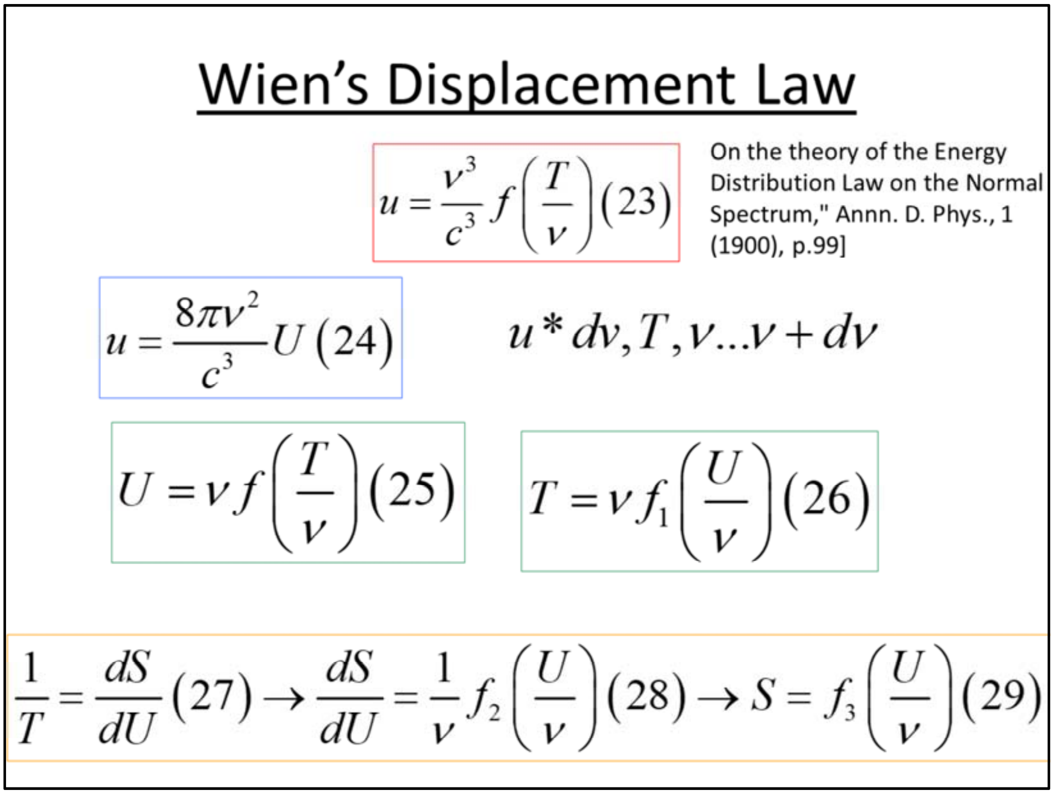
$$S_N = kN\left\{\left(1+\frac{P}{N}\right)\ln N\left(1+\frac{P}{N}\right) - \ln N - \frac{P}{N}\ln N\frac{P}{N}\right\} =$$

$$= kN\left\{\left(1+\frac{U}{\varepsilon}\right)\ln\left(1+\frac{U}{\varepsilon}\right) - \frac{U}{\varepsilon}\ln\frac{U}{\varepsilon}\right\}$$

$$\boxed{S_N = k\left\{\left(1+\frac{U}{\varepsilon}\right)\ln\left(1+\frac{U}{\varepsilon}\right) - \frac{U}{\varepsilon}\ln\frac{U}{\varepsilon}\right\} \quad (22)}$$

Planck's working hypothesis is the following: for N resonators to have the UN's vibrational energy together, the W probability must be proportional to the R number of all possible complexes formed by the distribution of UN energy between N resonators. The fact that this hypothesis is true can only be demonstrated through experience.

According to this hypothesis, we have the relationship (22).



Wien’s Displacement Law

$$\boxed{u = \frac{\nu^3}{c^3} f\left(\frac{T}{\nu}\right)(23)}$$

On the theory of the Energy Distribution Law on the Normal Spectrum," Annn. D. Phys., 1 (1900), p.99]

$$\boxed{u = \frac{8\pi\nu^2}{c^3} U\,(24)} \qquad u^* d\nu, T, \nu \ldots \nu + d\nu$$

$$\boxed{U = \nu f\left(\frac{T}{\nu}\right)(25)} \qquad \boxed{T = \nu f_1\left(\frac{U}{\nu}\right)(26)}$$

$$\boxed{\frac{1}{T} = \frac{dS}{dU}(27) \rightarrow \frac{dS}{dU} = \frac{1}{\nu} f_2\left(\frac{U}{\nu}\right)(28) \rightarrow S = f_3\left(\frac{U}{\nu}\right)(29)}$$

Wien's displacement law, also known as Kirchoff's theorem, establishes the proportionality between the radiated power and the absorbed power in the case of a heated body.

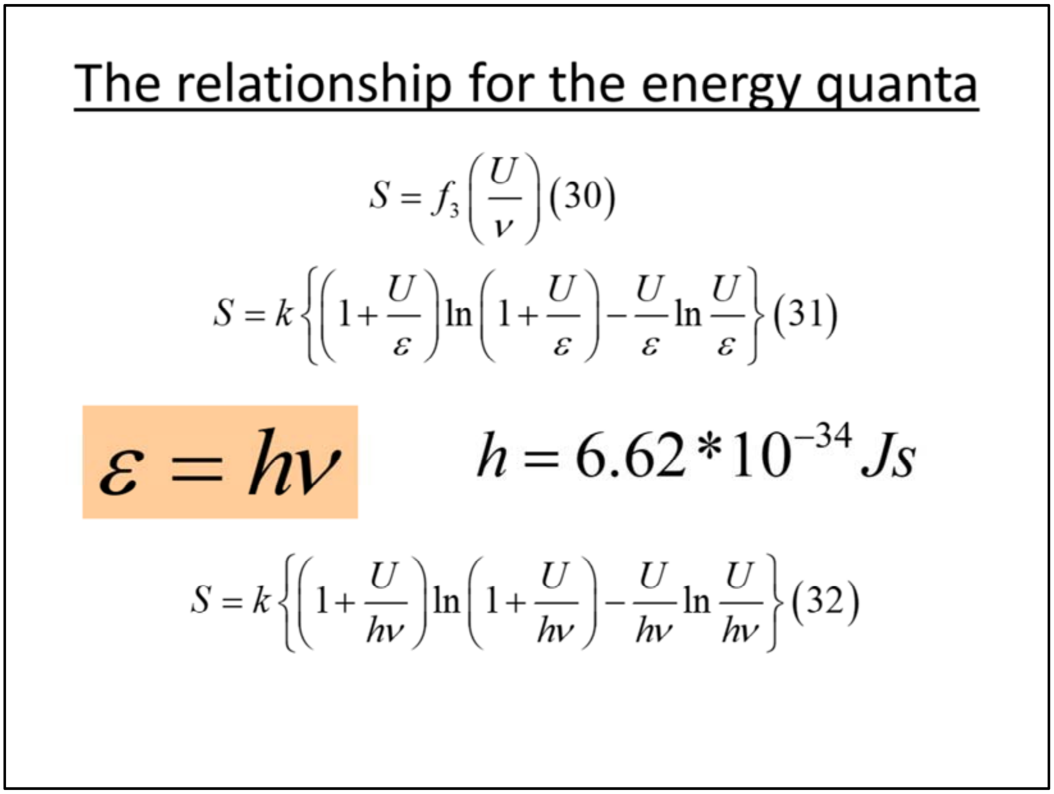
Thus, if we denote the density of the energy radiated by a body at the temperature T in the spectral range between ν and ν + dν, then we have the relation (23).

Plank demonstrated in an article [Planck M., "On the theory of the Energy Distribution Law on the Normal Spectrum," Annn. D. Phys., 1 (1900), p.99] that between the energy density u and the energy U of a stationary resonator, which is in a radiation field, and vibrating with the same frequency ν, there is the relation (24).

From the two relations (23) and (24) result the relation (25), respectively from (25) results (26).

On the other hand, from the relation (27), which expresses the connection between the entropy and the temperature of a thermodynamic system, results in relation (28), respectively relationship (29).

The relationship (29) shows that the entropy of the resonator depends only on the variable (U / ν), containing only the universal constant. This is the simplest form of Wien's displacement law.



The relationship for the energy quanta

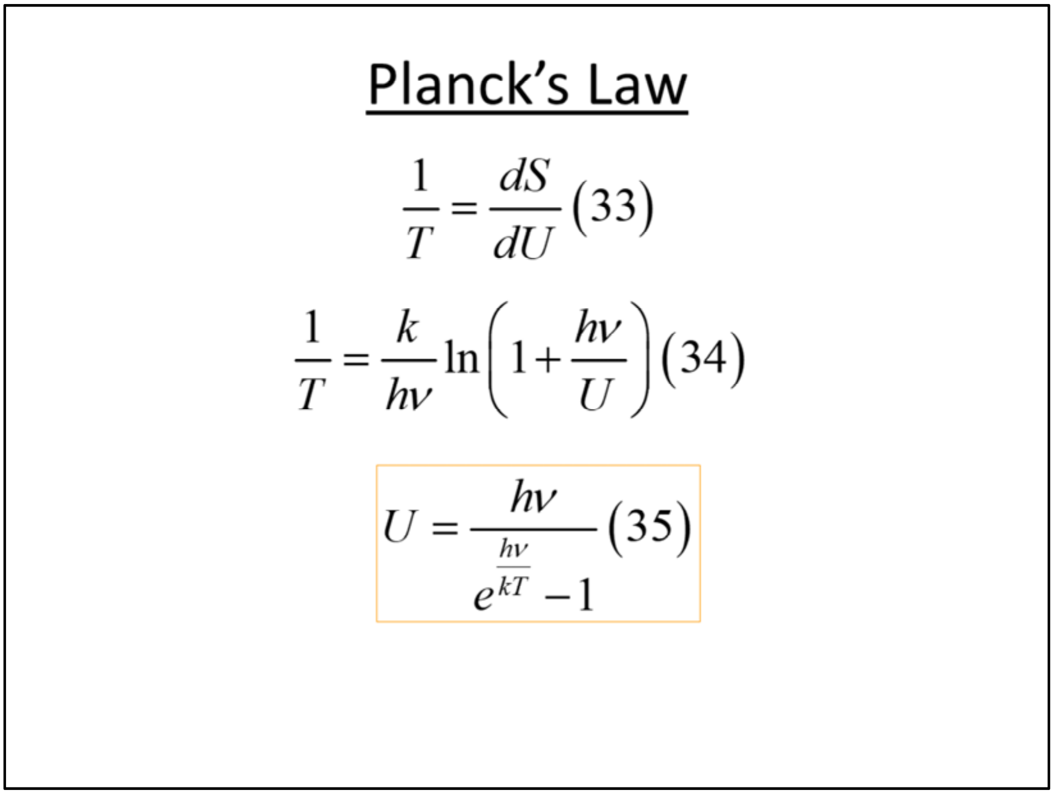
$$S = f_3\left(\frac{U}{\nu}\right) \quad (30)$$

$$S = k\left\{\left(1+\frac{U}{\varepsilon}\right)\ln\left(1+\frac{U}{\varepsilon}\right)-\frac{U}{\varepsilon}\ln\frac{U}{\varepsilon}\right\} \quad (31)$$

$$\varepsilon = h\nu \qquad h = 6.62 * 10^{-34}\, Js$$

$$S = k\left\{\left(1+\frac{U}{h\nu}\right)\ln\left(1+\frac{U}{h\nu}\right)-\frac{U}{h\nu}\ln\frac{U}{h\nu}\right\} \quad (32)$$

If we compare the relation (30) (the law of Wien's displacement) with the entropy relation (31), we can find that the expression of the energy quanta must be proportional to the frequency ν, and consequently we have the expression (32) for entropy .



Planck's Law

$$\frac{1}{T} = \frac{dS}{dU} \quad (33)$$

$$\frac{1}{T} = \frac{k}{h\nu} \ln\left(1 + \frac{h\nu}{U}\right) \quad (34)$$

$$\boxed{U = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1} \quad (35)}$$

From the relation (33), using the expression of the entropy (32) and of the radiation quanta we obtain the relation (34), equivalent to the relation (35).

The relation (7) represented the law of radiation and absorption of energy by a body at the temperature T, at the frequency v.

This law was rediscovered by Einstein, using considerations of quantum mechanics, in 1916.

Different forms of Planck's Law

$$U = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$

$$h\nu \gg kT$$

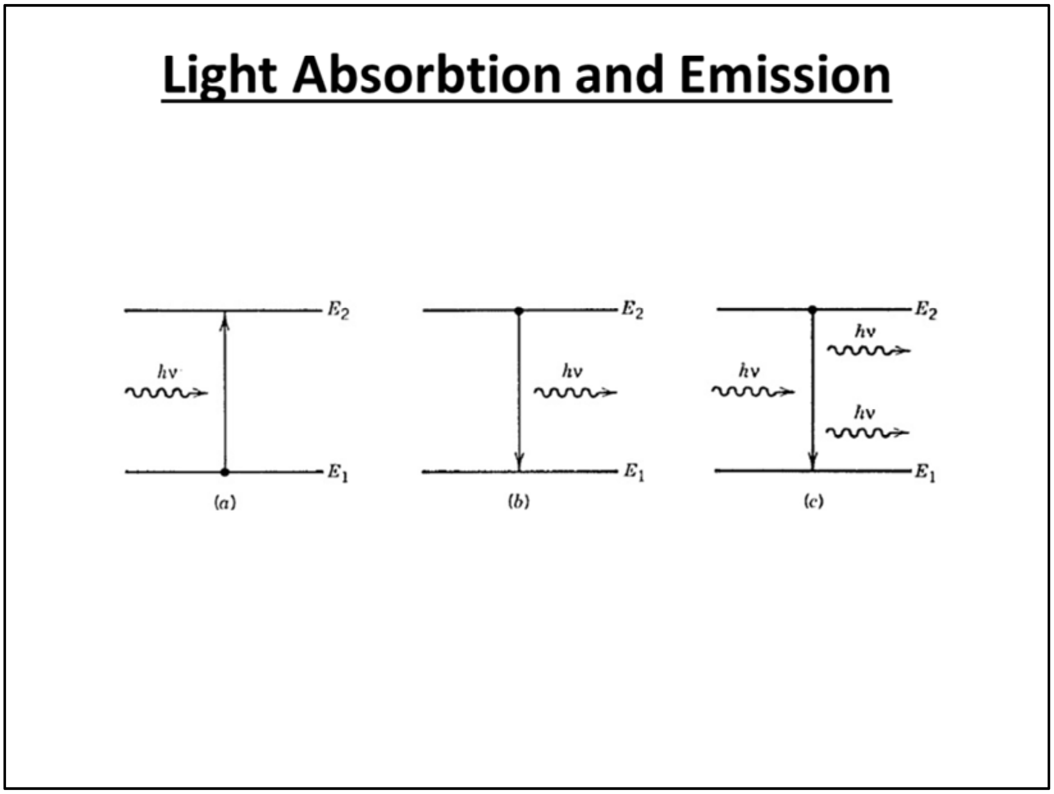
$$\frac{1}{e^{\frac{h\nu}{kT}} - 1} \approx e^{-\frac{h\nu}{kT}}$$

Low of Wien

$$h\nu \ll kT$$

$$e^{\frac{h\nu}{kT}} \approx 1 + \frac{h\nu}{kT}$$

Low
Rayleigh–Jeans



Light Absorbtion and Emission

Under normal conditions, all materials absorb light in the same maner they emit it.

The absorption process can be understood by understanding that the energies E1 and E2 correspond to the basic state and the excited state of the atoms in the absorbent material. If the energy of a photon, h * ν, of incident frequency light n is approximately equal to the energy difference Eg = E2-E1, the photon is absorbed by the atom, it passes into the excited state. Incident light is absorbed due to many such absorbing events taking place inside the environment.

Excited atoms sometimes return to their basic state and emit light.

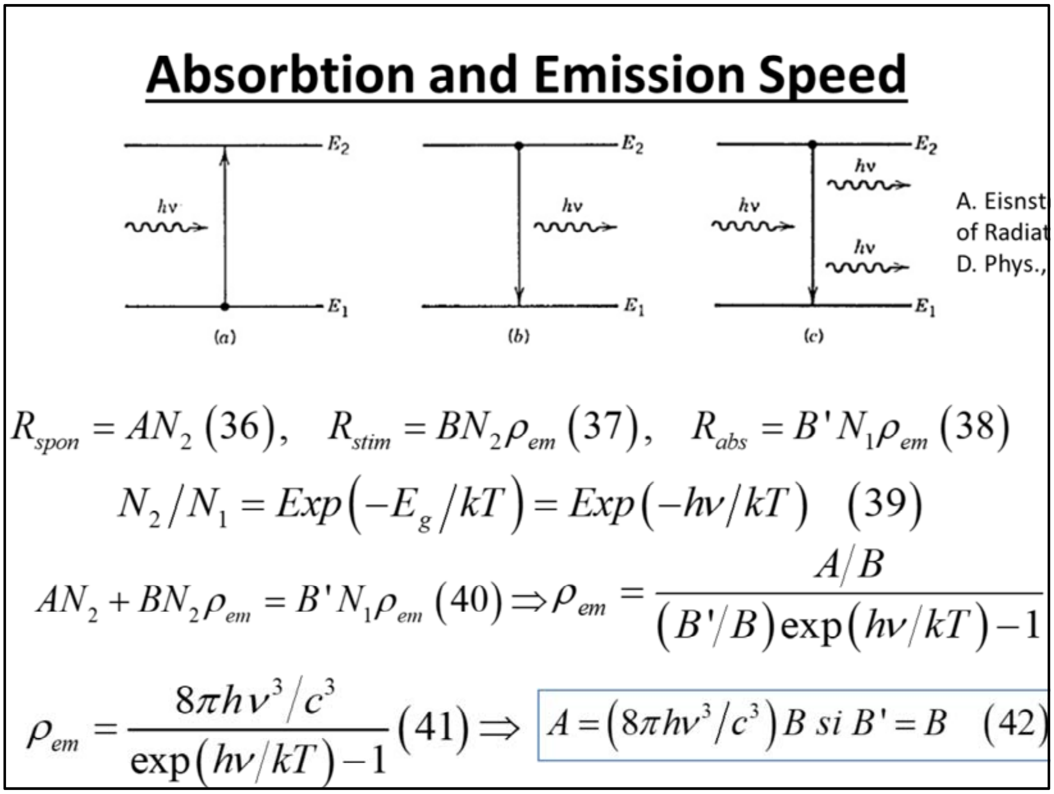
This light emission can take place through two fundamental processes: spontaneous emission and stimulated emission.

In case of spontaneous emission, photons are emitted in random directions with phases that are not correlated with each other.

In the case of stimulated emission, the emission is initiated by an existing photon. The photon emitted is identical to the original photon, both in energy (so frequency) and in phase and direction.

Lasers emit light through the stimulated emission process; the light emitted by them is said to be coherent.

LEDs emit light through the spontaneous emission process.



Absorbtion and Emission Speed

$$R_{spon} = AN_2 \;(36), \quad R_{stim} = BN_2\rho_{em} \;(37), \quad R_{abs} = B'N_1\rho_{em} \;(38)$$

$$N_2/N_1 = Exp\left(-E_g/kT\right) = Exp\left(-h\nu/kT\right) \quad (39)$$

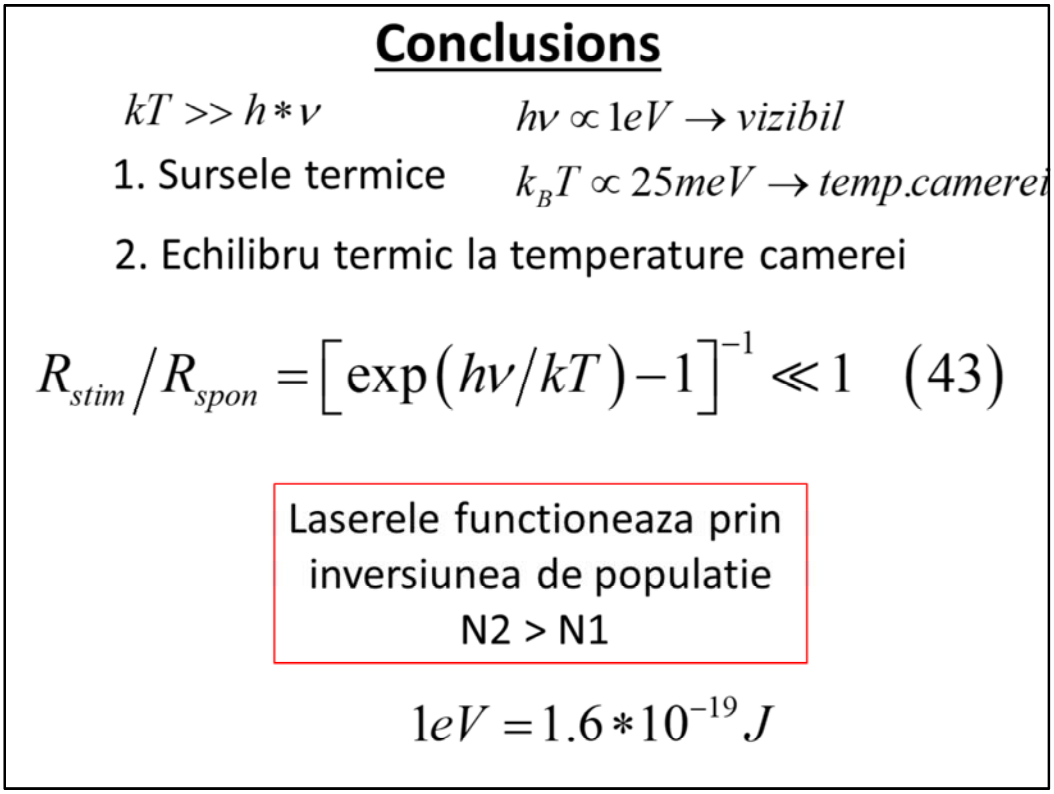
$$AN_2 + BN_2\rho_{em} = B'N_1\rho_{em} \;(40) \Rightarrow \rho_{em} = \frac{A/B}{\left(B'/B\right)\exp\left(h\nu/kT\right)-1}$$

$$\rho_{em} = \frac{8\pi h\nu^3/c^3}{\exp\left(h\nu/kT\right)-1}\;(41) \Rightarrow \boxed{A = \left(8\pi h\nu^3/c^3\right)B \;si\; B' = B \quad (42)}$$

First of all we consider a two-level atomic system that interacts with an electromagnetic field through transitions like those in the figure. If N1 and N2 are the densities of atoms in the basic and excited states respectively, and ρ_em (ν) is the spectral energy density of the electromagnetic field, the spontaneous emission, stimulated emission and absorption rates are described by the relations (36) - (38), where A, B and B 'are constant. At thermal equilibrium, atomic densities are distributed according to Boltzmann statistics (39). Since N1 and N2 do not change over time, in a state of thermal equilibrium, the up and down transition rates, on the energy scale, must be equal, which leads to (40).

On the other hand, at the thermal equilibrium, ρ_em must be identical to the spectral density of the black body radiation given by Planck's formula (41). It follows the relationships (42) between the constants.

These were obtained by Einstein [A. Eisnstein, Emission and Absorption of Radiation in Quantum Theory, Ann. D. Phys., 18, 1916, pp. 318-323]; for this reason, A and B are called Einstein's coefficients.



Conclusions

$kT >> h*\nu$ $h\nu \propto 1eV \rightarrow vizibil$

1. Sursele termice $k_B T \propto 25meV \rightarrow temp.camerei$

2. Echilibru termic la temperature camerei

$$R_{stim}/R_{spon} = \left[\exp\left(h\nu/kT\right)-1\right]^{-1} \ll 1 \quad (43)$$

Laserele functioneaza prin inversiunea de populatie
N2 > N1

$$1eV = 1.6*10^{-19} J$$

1. R_spon can be much grater than R_stim and R_abs, provided that KT> h * ν. Thermal sources work in this regime.

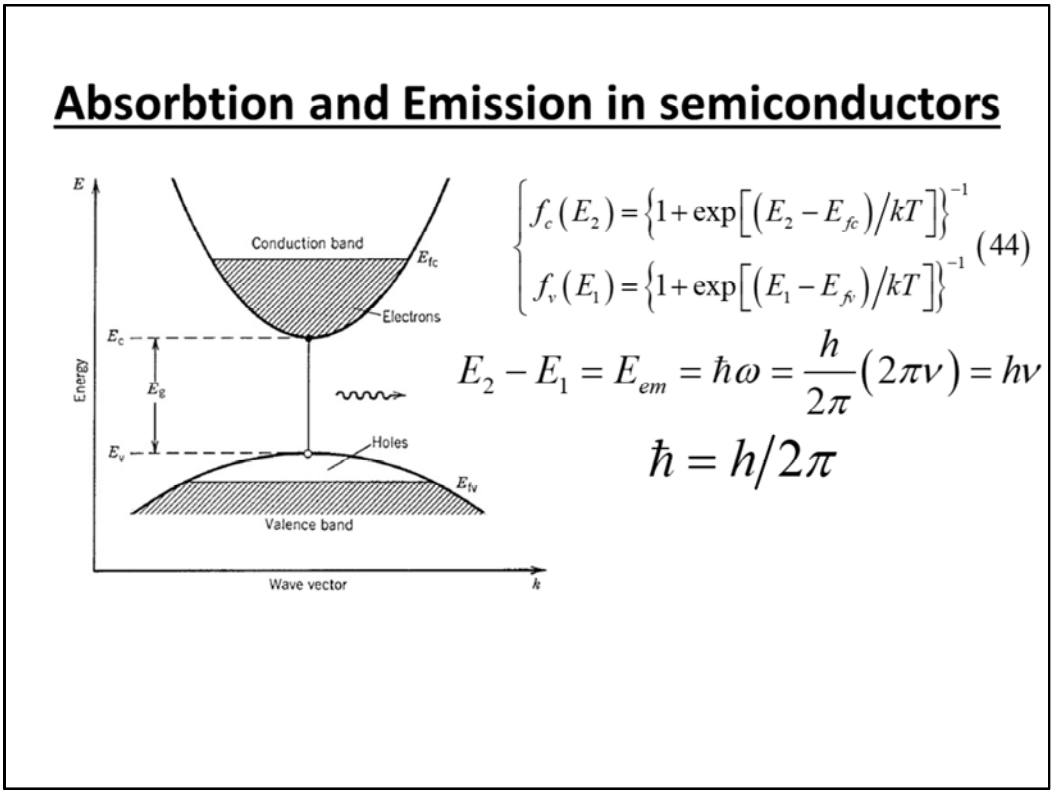
For near-visible or infrared radiation (h * ν ~ 1 eV), spontaneous emission always dominates the stimulated emission, under conditions of thermal equilibrium at room temperature (kT ~ 25 meV), the relationship (43) is verified.

From the relationship (43) it results that the lasers must operate away from a state of equilibrium. This is done by pumping the laser with an external energy source.

Even in the case of external atomic mass pumping, stimulated emission may not be dominant because it has to compete with the absorption process.

R_stim can only exceed R_abs if N2> N1. This condition represents the inversion of the population; condition that can not be achieved at thermal equilibrium.

In atomic systems, this population inversion is accomplished using 3 or 4 level pumping schemes, so that the external energy source passes the atomic population from the basic state to the excited state, which has a higher energy than E2.



Absorbtion and Emission in semiconductors

$$\begin{cases} f_c(E_2) = \left\{1+\exp\left[\left(E_2 - E_{fc}\right)/kT\right]\right\}^{-1} \\ f_v(E_1) = \left\{1+\exp\left[\left(E_1 - E_{fv}\right)/kT\right]\right\}^{-1} \end{cases} \quad (44)$$

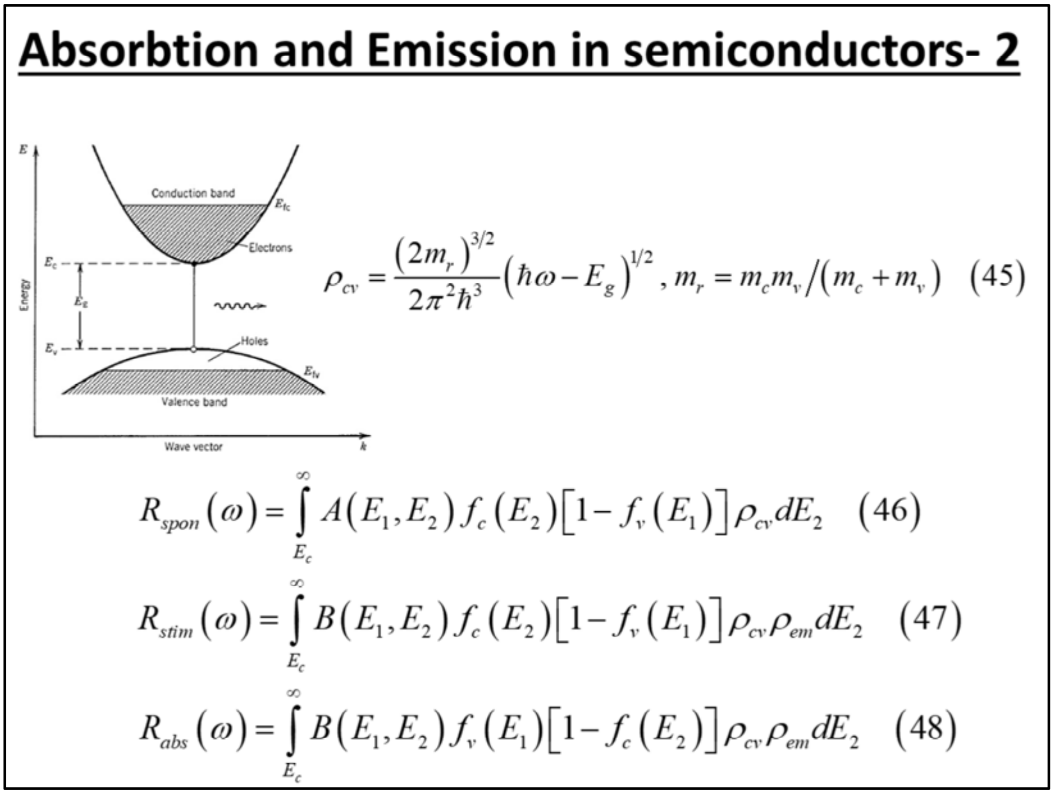
$$E_2 - E_1 = E_{em} = \hbar\omega = \frac{h}{2\pi}\left(2\pi\nu\right) = h\nu$$

$$\hbar = h/2\pi$$

Semiconductor emitting and absorbing have to be explaned using the energy bands model.

Spontaneous emission can only occur if the E2 energy state is occupied by an electron, and the E1 energy state is empty (i.e. occupied with a hole).

The probability that an electron occupies a level of energy in the conduction band or the valence band is given by the Fermi-Dirac distribution, the relationships (44), Efc and Efv being Fermi levels.



Absorbtion and Emission in semiconductors- 2

$$\rho_{cv} = \frac{(2m_r)^{3/2}}{2\pi^2\hbar^3}\left(\hbar\omega - E_g\right)^{1/2},\ m_r = m_c m_v/(m_c + m_v) \quad (45)$$

$$R_{spon}(\omega) = \int_{E_c}^{\infty} A(E_1, E_2) f_c(E_2)\left[1 - f_v(E_1)\right]\rho_{cv} dE_2 \quad (46)$$

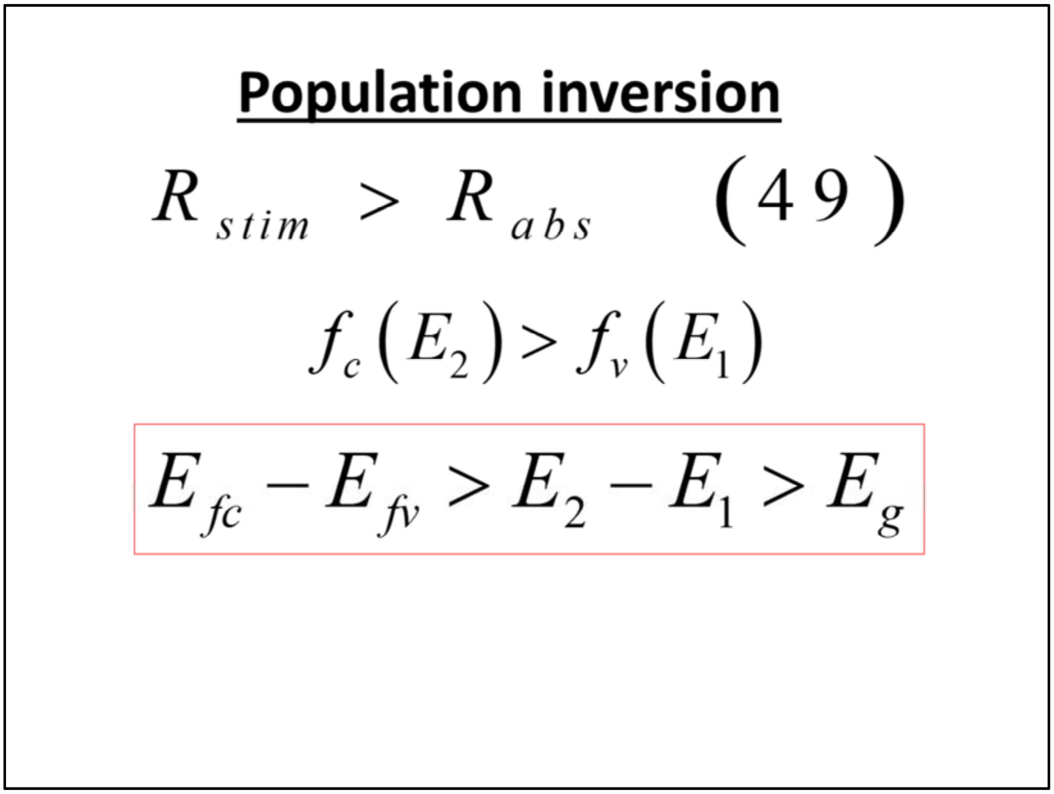
$$R_{stim}(\omega) = \int_{E_c}^{\infty} B(E_1, E_2) f_c(E_2)\left[1 - f_v(E_1)\right]\rho_{cv}\rho_{em} dE_2 \quad (47)$$

$$R_{abs}(\omega) = \int_{E_c}^{\infty} B(E_1, E_2) f_v(E_1)\left[1 - f_c(E_2)\right]\rho_{cv}\rho_{em} dE_2 \quad (48)$$

The total spontaneous emission rate, Rspon, at the frequency ω, is obtained summing up all the possible transitions between the two bands, for which E2-E1 = Eem = h * ν. Eem is the energy of the photon emitted. The result is (46).

ρ_cv is the density of states, defined as the number of states per volume unit and per unit energy interval, its expression being (45). In this relation, Eg is the forbidden band, and m_r is the reduced mass, where m_c and m_v are the effective masses of the electrons and holes in the conduction band and the valence band, respectively. p_cv does not depend on E2, but A depends on E2.

The analogous relations for stimulated emission and absorption are (47) and (48), where ρ_em is the spectral density of the photons.



Population inversion

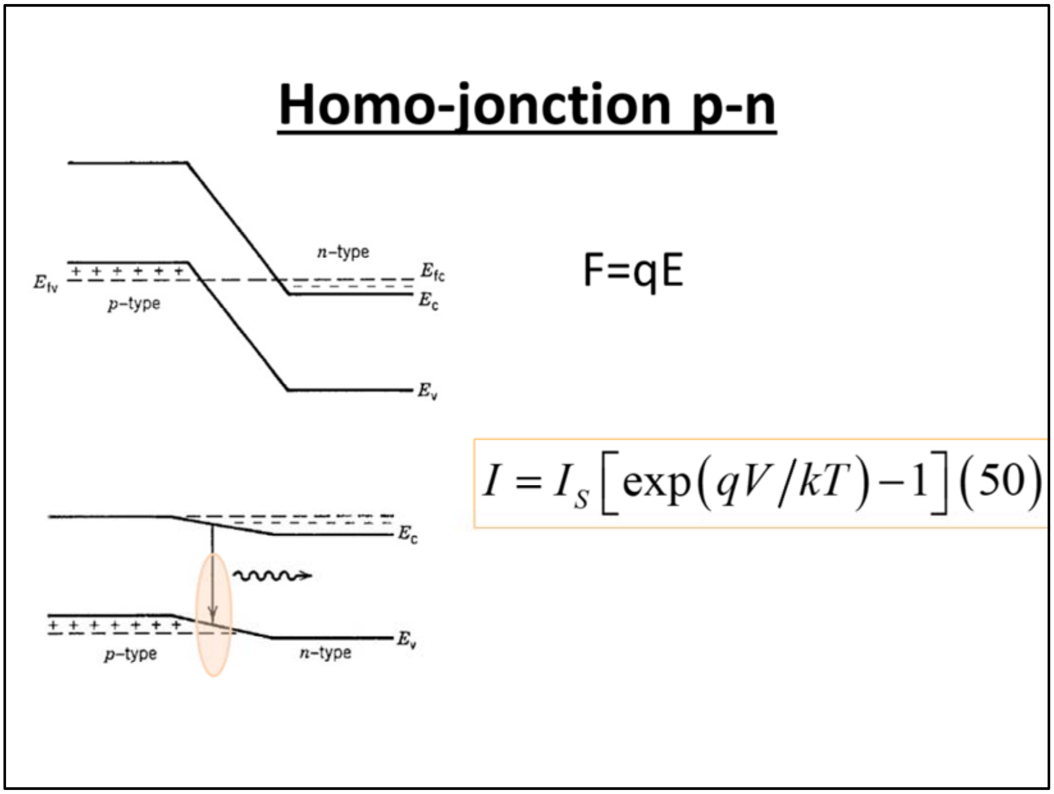
$$R_{stim} > R_{abs} \qquad (49)$$

$$f_c(E_2) > f_v(E_1)$$

$$\boxed{E_{fc} - E_{fv} > E_2 - E_1 > E_g}$$

Population inversion is obtained when we are satisfied with the relationship (49).

Since the minimum value of the E2 - E1 difference is Eg, the separation between Fermi levels must be greater than the forbiden band as the population inversion to occur. At thermal equilibrium, the two Fermi levels coincide (Efc = Efv). They can be separated by pumping energy into the semiconductor from an external source. The most convenient way to do this is to use a polarized direct junction.



Homo-jonction p-n

F=qE

$$I = I_S\left[\exp\left(qV/kT\right)-1\right](50)$$

Any optical light source is based on a p-n jonction. A semiconductor is made by type n by doping with impurities whose atoms have 1 valence electron more than the semiconductor atoms. For p, they have 1 electron less. In the case of the n-type semiconductor, the electrons in excess occupy the conduction states, normally empty of electrons in the undoped semiconductor (also called the intrinsic semiconductors). Fermi level, EFC, which is in the middle of the forbiden band of the semiconductor intrinsic, moving toward the conduction band with the increasing concentration of dopant. In highly doped semiconductors, the Fermi level is within the conduction band; such a semiconductor is said to be degenerate.

Similarly, the Fermi level, Efv, moves to the valence band for a p-type semiconductor and is inside it for highly doped semiconductors.

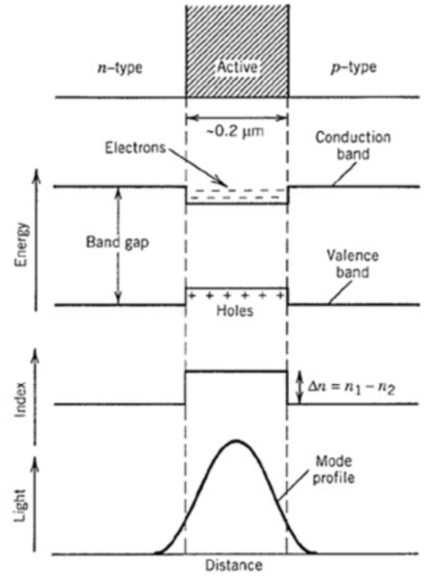
At thermal equilibrium, the Fermi level must be a continuous line along the p-n junction. This is achieved by diffusion of electrons and holes along the junction. The electrically charged impurities left behind result in the occurrence of an electric field that opposes the continued diffusion of electrons and holes in the thermal state of equilibrium.

The figure shows a diagram of energy bands for a thermally balanced junction that is under direct polarization (non-equilibrium).

When a p-n junction is polarized directly by applying an external voltage, the internal field is reduced. This reduction process reopen diffusion of electrons and holes along the junction . Thus the junction begins to circulate an electric current due to diffusion carriers. This current, I, exponentially increases with the applied voltage V, according to the relationship (50). Is is the saturation current that depends on the diffusion coefficients associated with electrons and voids. As shown in the figure, a region around the junction (known as the depleted region), electrons and holes are present at the same time when the p-n junction is forward-biased. These electrons and voids can be recombined by spontaneous or stimulated emission and generate light in an optical source.

The junction p-n of the figure is called homo-junction because the same semiconductor material is used by both sides of the junction. The problem with homojunction is that electron-hole recombination takes place in a relatively broad region (~ 1-10 microns) as determined by the length of the diffusion of electrons and holes. Since carriers are not confined in the immediate vicinity of the junction, it is difficult to achieve high density of carriers.

Hetero-junction p-n
Simultaneously Confinemant of Curriers and Fields



This problem of the confinement of the carriers can be solved by interposing a third thin layer, between the p-type and the n-type layers, which has a bandwidth less than the layers surrounding it. This intermediate layer may be intrinsic or doped.

The role of this additional layer is to confine injected carriers inside it at a direct polarization. Carriers confinement occurs as a result of the forbidden band discontinuity at the junction between two semiconductors having the same crystal structure (the same network constant) but different forbidden bands. Such junctions are called heterojunctions, and the devices are called double heterostructures.

Because the thickness of the intermediate layer can be controlled externally (typically ~ 0.1 microns), a high density of carriers can be achieved at a given injected current.

Using a heterojunction in light sources is advantageous for two reasons.

1) The difference between the forbidden bands of the two semiconductors helps to constrain the wearers in the intermediate layer, also called the active layer, because light is generated inside or as a result of electron-empty recombination.
2) The active layer also has a slightly higher refractive index than the surrounding p and n layers, for the simple reason that it has a smaller forbidden band. Therefore, the active layer acts as a dielectric guide and will support optical modes whose number can be controlled by changing the thickness of the active layer (similar to the modes supported by an optical fiber). It is essential that the heterostructure also confines the light generated in the active layer due to its higher index of refraction.

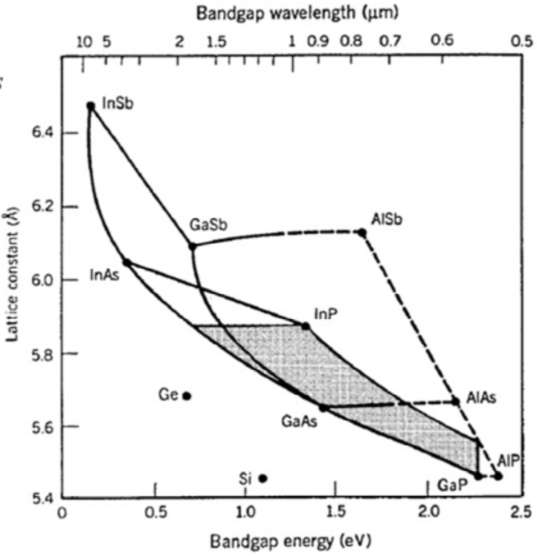
In the figure there is schematically illustrated the simultaneous confinement of the load carriers and the light in the active region of a heterostructure.

Hetero - semiconductors

$$h = 6.626 * 10^{-34}\, Js$$

$$c = 3 * 10^{8}\, m/s$$

$$1eV = 1.6 * 10^{-19}\, J$$



Ternary
$Al_xGa_{1-x}As$

Quaternary
$In_{1-x}Ga_xAs_yP_{1-y}$
$x/y = 0.45$

$$E_g = h\nu = h\frac{c}{\lambda}$$

$$E_g(x) = 1.424 + 1.247x, (0 < x < 0.45) \quad (51)$$

$$E_g(y) = 1.35 - 0.72y + 0.12y^2, (0 < y < 1) \quad (52) \qquad x/y = 0.45$$

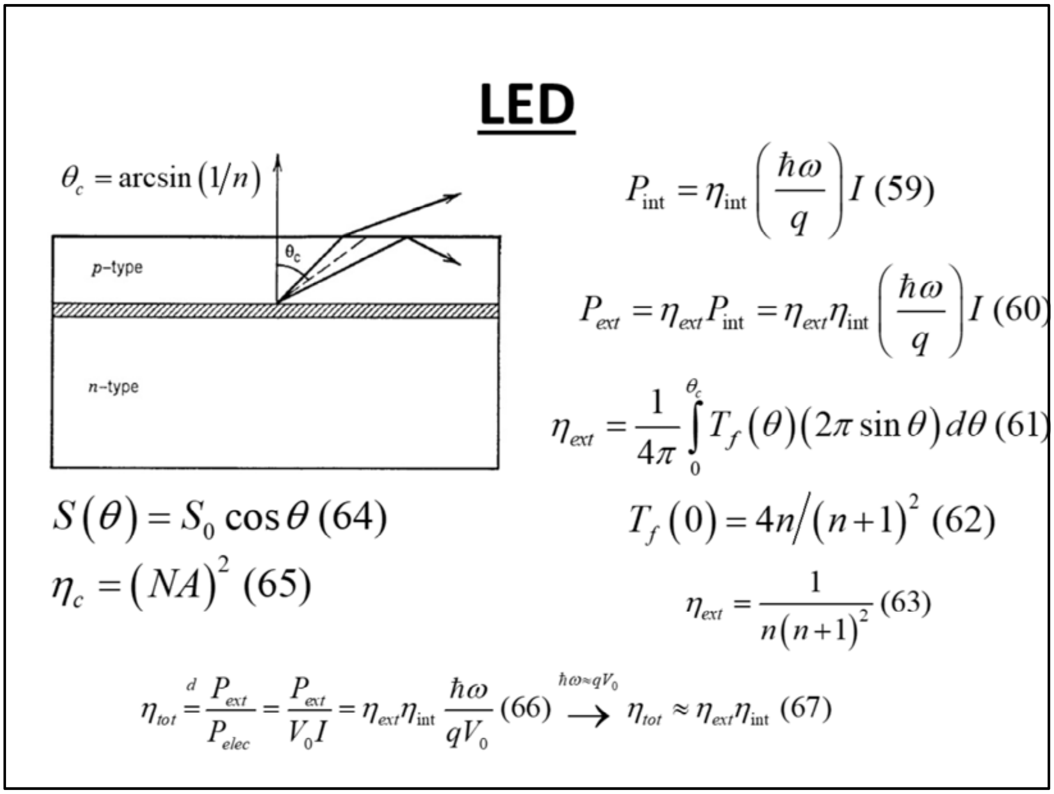
Almost any direct semiconductor can be used to make a homo-junction able to emit light by spontaneous emission. The choice is considerably limited, however, in the case of heterostructures, because their performance depends on the quality of the heterojunction interface between two semiconductors with different bands. In order to reduce network defect formation, the network constannts of the two materials must be different by no more than 0.1%. Nature does not provide semiconductors with network constants that are equal with such precision. However, they can be made by forming ternary or quaternary compounds, where a fraction of the network locations in a natural binary semiconductor (eg, GaAs) is replaced by other elements. In the case of GaAs, a ternary AlxGal-xAs compound can be accomplished by replacing a part of Ga atoms with Al atoms. The semiconductor thus obtained has almost the same network constant, but a larger forbidden band. The forbidden band depends on the fraction x and it can be approximated by a linear relationship (51), where Eg is expressed in eV.

In the figure, the points are binary semiconductors, and the lines between them correspond to ternary semiconductors. The dotted part of the line indicates that the ternary compound is indirect. The area of a closed polygon corresponds to quaternary compounds. Their band is not necessarily direct. The area has a series of ternary and quaternary direct-line components, formed using Indiu, Galiu, Arsen and Posfor.

The horizontal line that binds GaAs and AlAs corresponds to the ternary compound AlxGal-xAs, which is a direct semiconductor for values of x smaller then 0.45. The active layer and the cladding layer are formed such that x is larger for the cladding compared to its value in the active layer. The wavelength of the light emission is determined by the forbidden band because the photon has an energy approximately equal to the forbidden band.

For example, the 0.87 micron wavelength is emitted by an active GaAs layer (Eg = 1.424 eV). The wavelength can be reduced to 0.81 microns using an active layer with x = 0.1. Optical sources based on GaAs typically emit in this range, and were used in the first generation of optical systems.

In the optical fiber chapter, we saw that it is preferable that the communication systems operate on lengths of 1.3-1.6 microns, where the dispersion and losses in fiber are considerably reduced compared to the 0.85 micron range. The semiconductor used for optical sources in this range is InP. As can be seen from the horizontal line passing through InP, the band can be considerably reduced by manufacturing quaternary compounds In1-xGaxAsyP1-y, while the network constant remains unchanged, that of InP. The fractions x and y can not be arbitrarily chosen, but in the ratio x / y = 0.45, to ensure the constanty of the network constant. The band of quaternary compounds can only be expressed by y, the relationship (52). The lowest forbidden band is obtained for y = 1. The corresponding ternary component is In0.5Ga0.45As careful light emitting around 1.65 microns (Eg = 0.75 eV). By an appropriate choice of fractions x and y, the In1-xGaxAsyP1-y source can operate in the range 1.0 - 1.65 microns.



LED

$$\theta_c = \arcsin\left(1/n\right)$$

$$P_{\text{int}} = \eta_{\text{int}}\left(\frac{\hbar\omega}{q}\right) I \ (59)$$

$$P_{ext} = \eta_{ext} P_{\text{int}} = \eta_{ext}\eta_{\text{int}}\left(\frac{\hbar\omega}{q}\right) I \ (60)$$

$$\eta_{ext} = \frac{1}{4\pi}\int_0^{\theta_c} T_f(\theta)\left(2\pi\sin\theta\right) d\theta \ (61)$$

$$T_f(0) = 4n/(n+1)^2 \ (62)$$

$$\eta_{ext} = \frac{1}{n(n+1)^2} \ (63)$$

$$S(\theta) = S_0\cos\theta \ (64)$$

$$\eta_c = (NA)^2 \ (65)$$

$$\eta_{tot} \overset{d}{=} \frac{P_{ext}}{P_{elec}} = \frac{P_{ext}}{V_0 I} = \eta_{ext}\eta_{\text{int}}\frac{\hbar\omega}{qV_0} \ (66) \xrightarrow{\hbar\omega\approx qV_0} \eta_{tot} \approx \eta_{ext}\eta_{\text{int}} \ (67)$$

In its simplest form, a LED is a homo-junction directly polarized. Radiativ recombination of electron-hole pairs in the active region generates light. Some of these escape from the device and can be plugged into an optical fiber. The emitted light is incoherent, with a relatively large spectral width (30 - 60 nm) and relatively large angular magnitude.

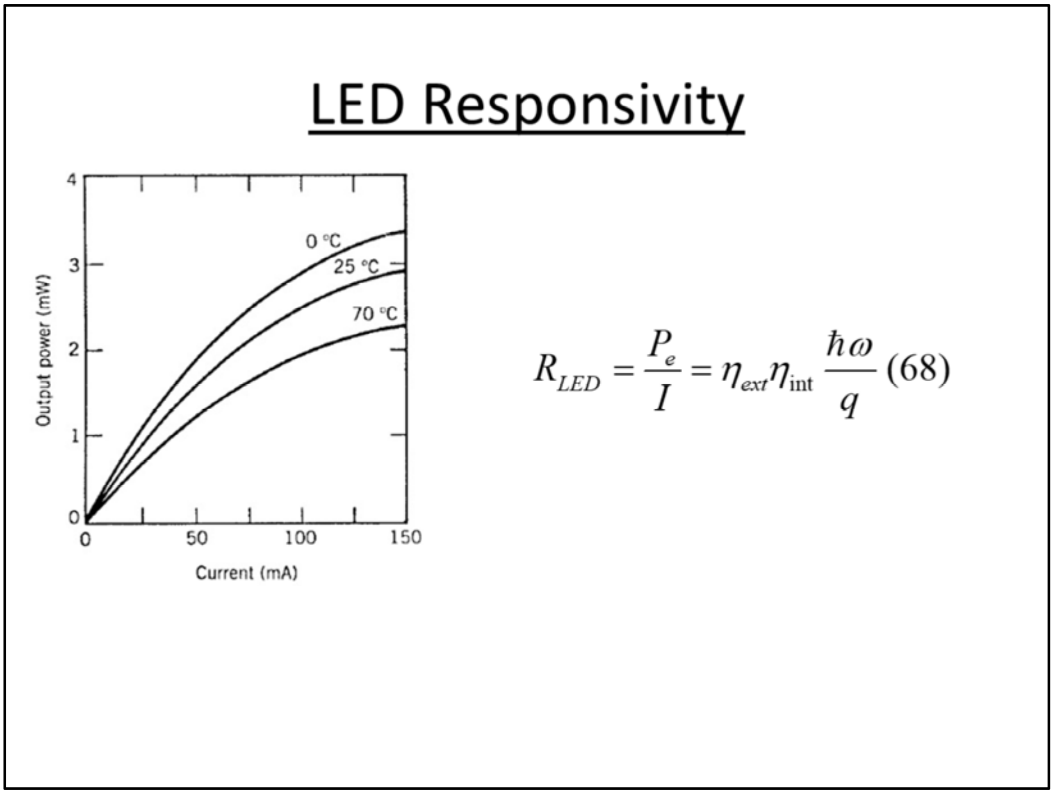
Power-Current characteristics

At a given current I, the injection rate of the carriers is I / q. In stationary mode, the electron-hole recombination velocity, through radiative and non-radiative processes, must be equal to the I / q carrier injection rate. Because the internal quantum efficiency determines the fraction of the electron-hole pairs that by recombination contributes to spontaneous emission, the photon generation speed will be η_int * I / q. The internal optical power will therefore be given by the relation (59), where h * ω is the energy of the photon. If η_ext is the fraction of the photons escaping from the device, the optical power emitted will be (60).

η_ext is the external quantum efficiency. It can be calculated taking into account internal absorption and total internal reflections at the semiconductor-air interface. Only light emitted in an angle cone θ_c escapes from the LED surface, n being the refractive index of the semiconductor. Internal absorption can be avoided by using LED heterostructures in which the cladding layers surrounding the active region, are transparent to the generated radiation. External quantum efficiency can be calculated with relation (61). The Fresnel's transmissivity depends on the angle θ. In the case of normal incidence (θ = 0), we have the relation (62). If we replace the relation (62) in relation (63), we obtain an approximation of η_ext, relation (63). For a typical value, n = 3.5, η_ext = 1.4%, which shows that only a small part of the internal power becomes useful outside.

Other losses in the useful power are produced at the coupling of the light emitted in the optical fiber. Because the emitted light is incoherent, the LED acts as a Lambertian source with an angular distribution given by eq.(64), where S_0 is the intensity in the direction θ = 0. The coupling efficiency of such a source is given by eq.(65). Since the numerical aperture for an optical fibers typically ranges from 0.1 to 0.3, only a small percentage of the emitted power is coupled to the fiber. Normally, the power released in the fiber by a LED is at most 100 µW, even though the internal power can easily exceed 10 mW.

A measure of the performance of an LED is the total efficiency, Eq. (66), defined as the ratio of the emitted optical power. Pext, and the electrical power consumed, Pelec, where P_elec = V0 * I, V_0 beeing the voltage drop on the LED.



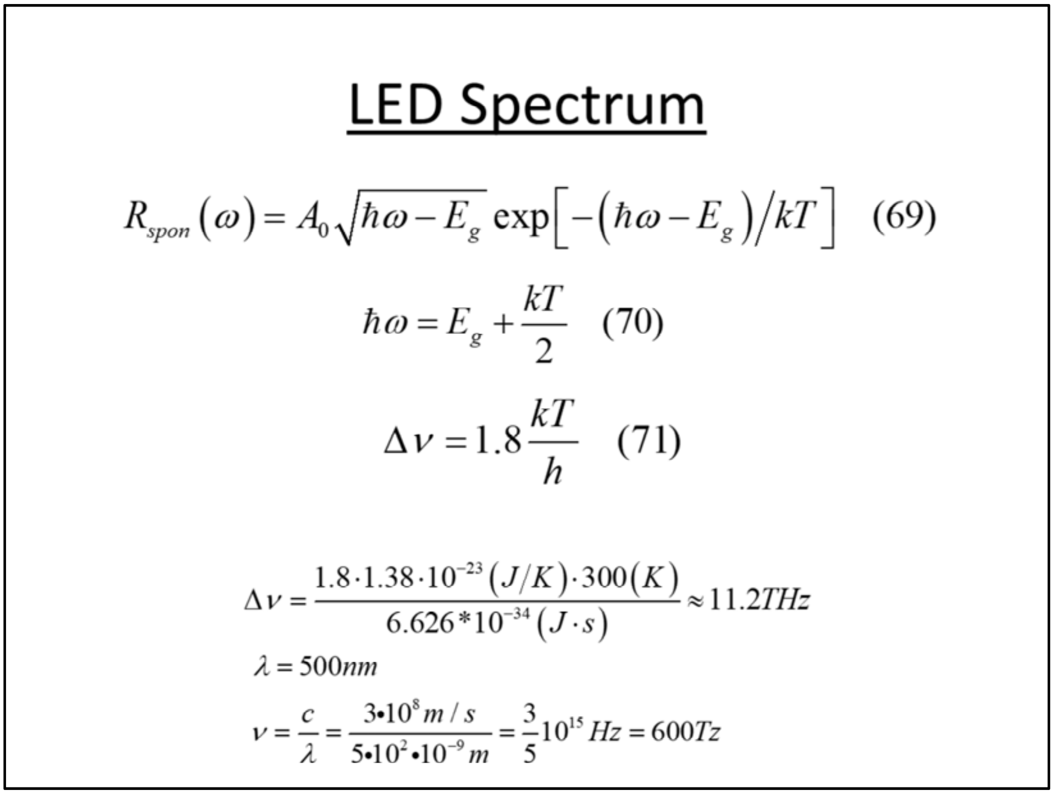
LED Responsivity

$$R_{LED} = \frac{P_e}{I} = \eta_{ext}\eta_{\mathrm{int}} \frac{\hbar\omega}{q} \quad (68)$$

Another dimension used in catalogs to characterize the performance of a LED is responsivity. It is defined in relation (68).

Typical responsivity is ~ 0.01 W / A. Responsivity remains constant as long as a linear relationship between Pe and I is maintained. The reason for decreasing responsivity is the increase in the temperature of the active region.

Internal quantum efficiency is generally temperature-dependent because we are witnessing an increase in the non-radiative recombination rate with the increasing temperature.



LED Spectrum

$$R_{spon}(\omega) = A_0\sqrt{\hbar\omega - E_g}\,\exp\left[-\left(\hbar\omega - E_g\right)/kT\right] \quad (69)$$

$$\hbar\omega = E_g + \frac{kT}{2} \quad (70)$$

$$\Delta\nu = 1.8\frac{kT}{h} \quad (71)$$

$$\Delta\nu = \frac{1.8\cdot 1.38\cdot 10^{-23}\left(J/K\right)\cdot 300\left(K\right)}{6.626*10^{-34}\left(J\cdot s\right)} \approx 11.2THz$$

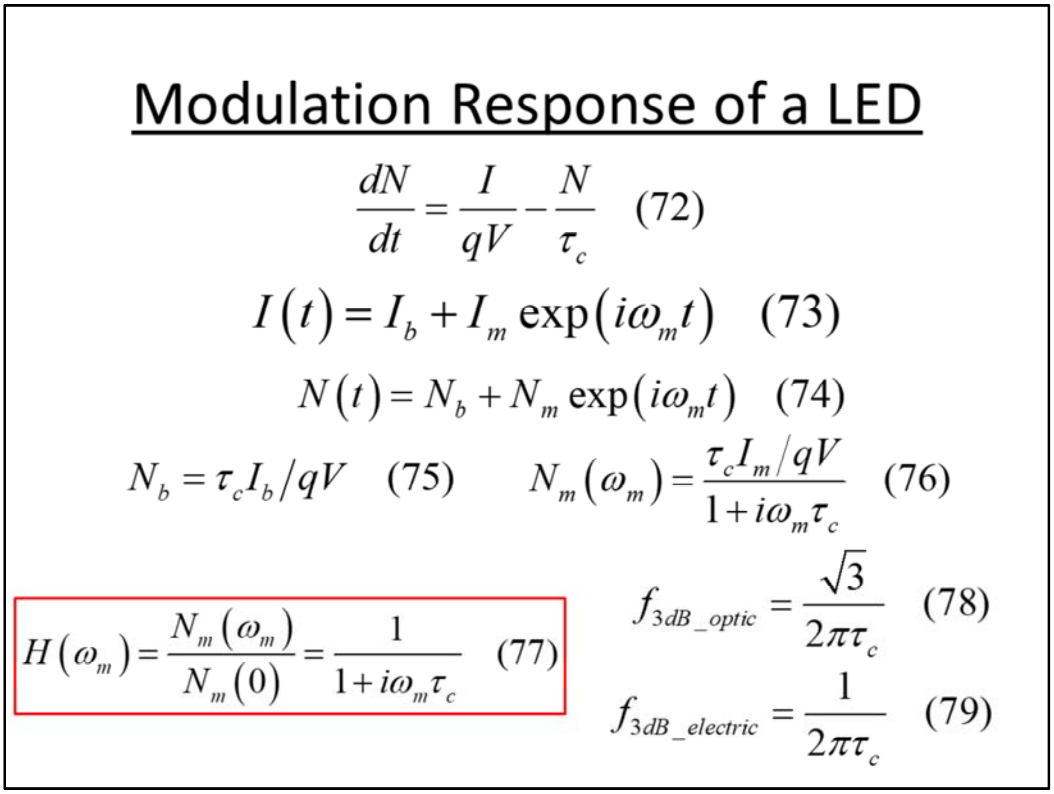
$$\lambda = 500nm$$

$$\nu = \frac{c}{\lambda} = \frac{3\bullet 10^{8}m/s}{5\bullet 10^{2}\bullet 10^{-9}m} = \frac{3}{5}10^{15}Hz = 600Tz$$

The spectrum of the LED is given by spontaneous emission spectrum R_spon (ω). In relation (69) we have an approximation of this spectrum, where A0 is a constant, and Eg is the forbidden band.

It is seen that the maximum of the spectrum is obtained for the relation (70), and the spectrum width at half of the maximum (FWHM) will be given by the relation (71). At room temperature (T = 300 K), the FWHM is around 11 THz.

Due to the large spectral width (Δλ = 50 - 60 nm), the bit rate is considerably limited by fiber dispersion when using an LED as the source: 10-100 Mb / s over several km distances.



Modulation Response of a LED

$$\frac{dN}{dt} = \frac{I}{qV} - \frac{N}{\tau_c} \quad (72)$$

$$I(t) = I_b + I_m \exp(i\omega_m t) \quad (73)$$

$$N(t) = N_b + N_m \exp(i\omega_m t) \quad (74)$$

$$N_b = \tau_c I_b / qV \quad (75) \qquad N_m(\omega_m) = \frac{\tau_c I_m / qV}{1 + i\omega_m \tau_c} \quad (76)$$

$$\boxed{H(\omega_m) = \frac{N_m(\omega_m)}{N_m(0)} = \frac{1}{1 + i\omega_m \tau_c} \quad (77)}$$

$$f_{3dB\_optic} = \frac{\sqrt{3}}{2\pi\tau_c} \quad (78)$$

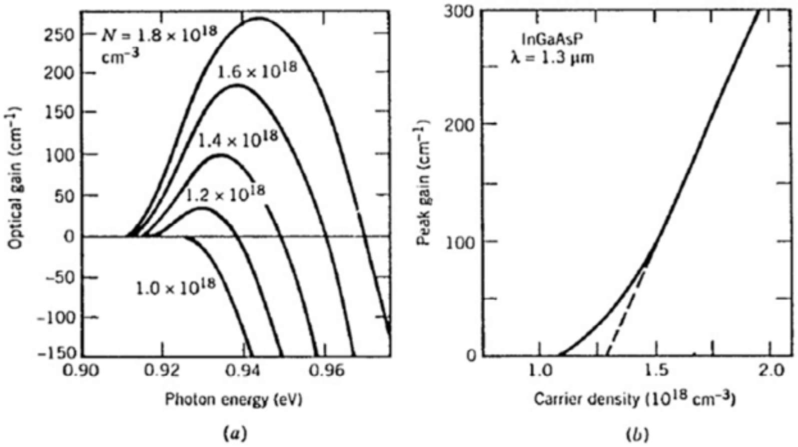
$$f_{3dB\_electric} = \frac{1}{2\pi\tau_c} \quad (79)$$

The modulation response of the LED depends on the dynamics of the carriers and is limited by the lifetime of the carriers, τ_c. This response can be deduced using a velocity equation for carrier density N. Since electrons and holes are injected into pairs and recombined in pairs, it is sufficient to consider the dynamic equation for a single type of carriers. This equation must include all the mechanisms by which electrons appear and disappear in the active region. It has the form of the relationship (72), in which the last term includes both processes: radiativ and non-radiativ.
We consider a sinusoidal modulator signal that modulates the injected current, the relationship (73), where Ib is the polarization current, Im is the modulation current and ω_m is the modulation frequency. Since equation (72) is linear, the general solution can be written in the form (74), where in constants are given in eqs. (75) and (76), V is the volume of the active region.
The modulating power, Pm, depends linear of Nm. Therefore, the LED's transfer function is defined by relation (77).
By analogy to fiber optic, the modulation band at 3-dB is defined as the modulation frequency at which $|Hm|$ decreases by 3 dB, or a factor equal to 2. It follows the relationship (78).
Typically, τ_c is in the range of 2 to 5 ns for LEDs on InGaAsP. The appropriate modulation band is between 50 - 140 MHz.
Note that the relationship (78) indicates the optical band because f_3dB is defined as the frequency at which the optical power is reduced by 3 dB (that is, by half).
The corresponding electrical band is frequent in which $|Hm|$^2 is reduced by 3 dB, that is given by the relationship (79).

LASER Diode

- Emits high power (~100 mW)
- Emits coherent light
- Light emited is collimated (coupling 50%)
- Spectral width is narrow (~ 10 Gb/s)
- Direct modulation of LASER until high frequencies (~ 25 GHz)

Optical Gain

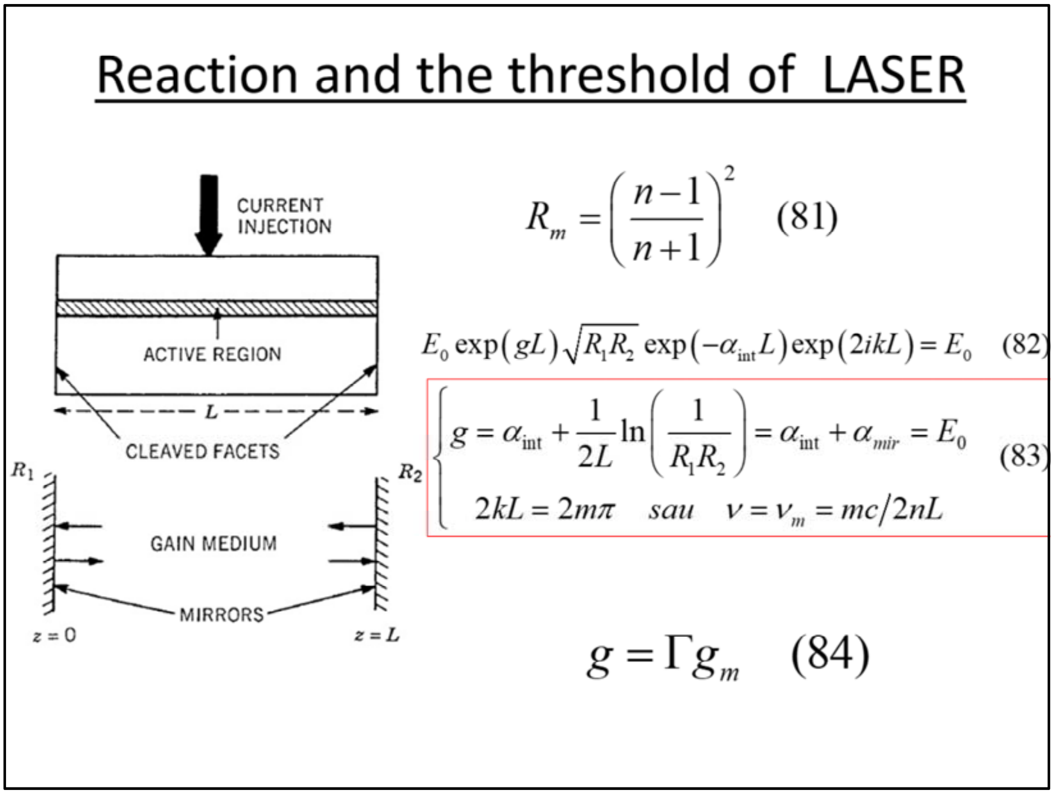


$$g_p\left(N\right) = \sigma_g\left(N - N_T\right) \quad (80)$$

Stimulated emission dominates only when the population inversion condition is satisfied. For semiconductor lasers, this condition is satisfied by doping the p-type and n type cladding layers very strong so that the separation between the Fermi levels is greater than the forbidden band, under direct polarization of the
P-n junction.
When the density of injected carriers in the active region exceeds a certain value, *called transparency value*, population inversion is achieved and the active region presents an *optical gain*: an input signal propagating inside this active layer will be amplified by exp(gz), where g is the gain coefficient. g is proportional to (R_stim - R_abs).
In Fig. (A) we see numerically calculated gain for an active InGaAsP layer at the 1.3 micron wavelength , for different values of the injected carrier density, N. For N = 1x10 ^ (18) cm ^ (-3) , g <0 because population inversion has not yet occurred. As N increases, g becomes positive in a spectrum that increases in turn with N. The value of the gain, g_p, also increases with N and moves towards higher energies. The variation of g_p with N is shown in Fig. (B). For N> 1.5x10 ^ (18) cm ^ (-3), g_p varies almost linearly with N. Figures show that the optical gain in semiconductors rises rapidly from the moment the population inversion was made. Because of this gain, semiconductor lasers can be manufactured with physical dimensions of less than 1 mm.

The almost linear dependence of g_p of N suggests the relation (80), in which N_T is the transparency value of carrier density and σ_g is the differential gain. Typical values for N_T and σ_g, for InGaAsP, are in the range 1.0 - 1.5x10 (18) cm (-3) and (2 – 3)x10-4 (- 16) cm2, respectively.

Semiconductor lasers with a higher value for σ_g perform better because the same value of winning can be achieved at a lower carrier density, which means at a lower injected current. In quantum hole semiconductor lasers, σ_g is typically twice as large.



Reaction and the threshold of LASER

$$R_m = \left(\frac{n-1}{n+1}\right)^2 \quad (81)$$

$$E_0 \exp(gL)\sqrt{R_1 R_2}\exp(-\alpha_{\text{int}} L)\exp(2ikL) = E_0 \quad (82$$

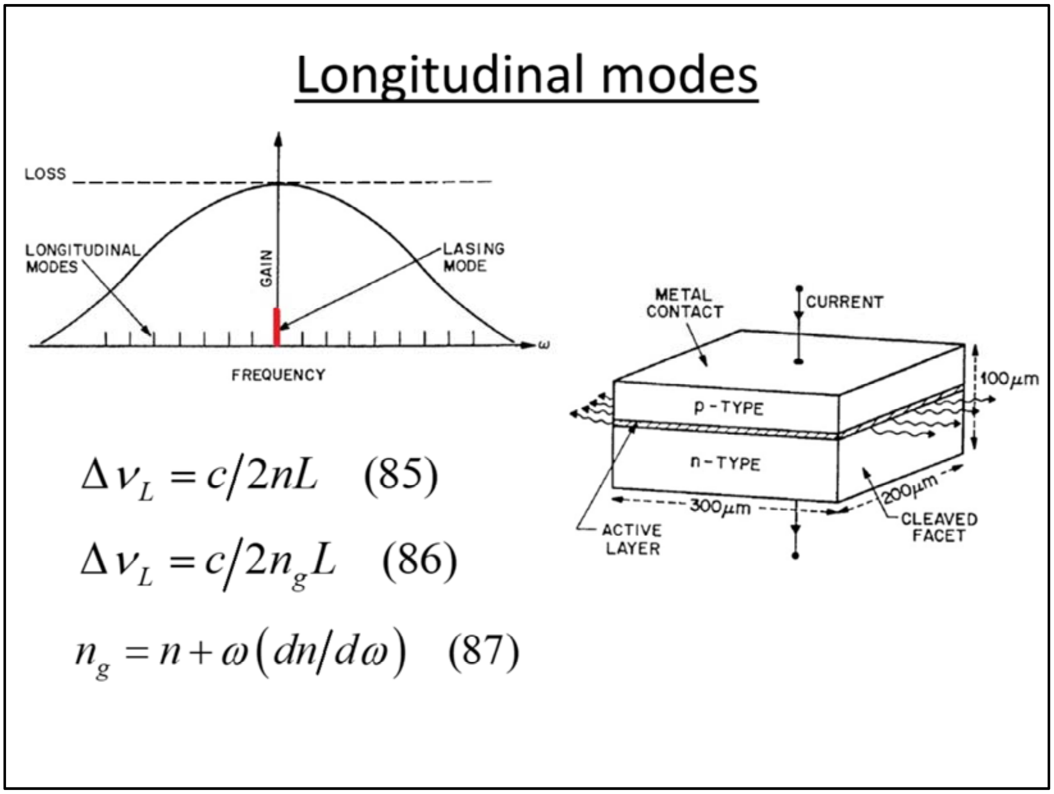
$$\begin{cases} g = \alpha_{\text{int}} + \dfrac{1}{2L}\ln\left(\dfrac{1}{R_1 R_2}\right) = \alpha_{\text{int}} + \alpha_{mir} = E_0 \\ 2kL = 2m\pi \quad sau \quad \nu = \nu_m = mc/2nL \end{cases} \quad (83$$

$$g = \Gamma g_m \quad (84)$$

Optical gain is not enough for laser operation. The other ingredient is the optical response, which turns the amplifier into the oscillator. In most lasers, the reaction is obtained by introducing the active medium into a Fabry-Perot cavity formed by the use of two mirrors. In the case of semiconductor lasers, the role of these mirrors is played by the two sides of the chip. The reflectivity of these mirrors is given by the relation (81), where n is the refractive index of the active medium. For example, for n = 3.5, the side‘s reflectivity is 30%. Although the FP cavity of the two cleaved planes has significant losses, the gain of the active medium is large enough for these losses to be tolerable.

The laser threshold concept can be understood by observing that some of the photons generated by stimulated emission are lost due to cavity losses and therefore must always be replaced. If the optical gain is not large enough to compensate for the loss of the cavity, the photon population can not be increased. Thus, a minimum gain is required for a laser to work. This minimum value is achieved by pumping the laser above the threshold level. The current required to reach the threshold is called *threshold current*.

To get the threshold condition a simple way is to analyse the amplitude change of a plane wave while traveling through the active region. Considering a plane of amplitude E_0, the frequency ω and the number of k = n * ω / c. During a return trip, the amplitude increases with exp (g / 2) (2L)] due to the gain (g is gaining power) and its phase is subdivided by 2kL, where L is the length of the laser cavity. At the same time, the amplitude changes with sqr (R1 * R2) * exp (-α_int * L) due to the reflection on the laser faces and due to the internal losses formed by the absorption of free carriers, impedance, etc.
In stationary mode, the plane wave must remain unchanged, which leads to the relationship (82). Drawing the module and phase in the two members, we get the relationships (83). Note that g is not the same as the gain of the material presented in fig. (A) of the slide preceding. Optical modes extend beyond the active region, while the win only exists within it. Therefore, we have the relation (84) in which gamma is the confinement factor of the active region, which has a typical value <0.4.



Longitudinal modes

$$\Delta \nu_L = c/2nL \quad (85)$$

$$\Delta \nu_L = c/2n_g L \quad (86)$$

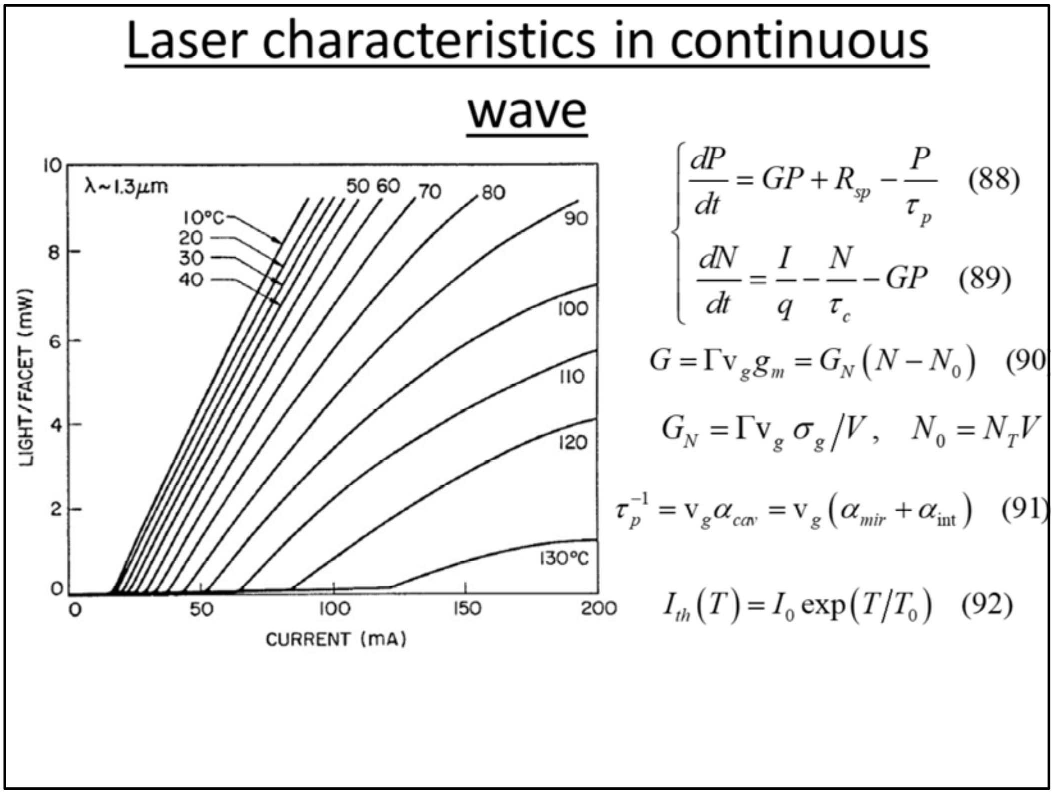
$$n_g = n + \omega\left(dn/d\omega\right) \quad (87)$$

The phase condition shows that the laser frequency ν must be equal to one of the frequencies ν_m, where m is an integer. These frequencies correspond to longitudinal modes and are determined by the optical length (n * L). The space between the longitudinal modes is constant, the relation (85), if the frequency dependence of the refractive index, n, is ignored. If the dispersion of material is included, then the spacing is given by the relation (86) in which n_g is the group refraction index, given by the relation (87). Typically, the spacing is 100-200 GHz for L = 200-400 microns.

A Fabry-Perrot cavity semiconductor laser emits light in the form of several longitudinal cavity modes. The gain spectrum g (ω) is wide enough (~ 10 THz) for several longitudinal modes of the FP cavity to present simultaneously the necessary gain.
The mode closest to the maximum gain becomes the dominant mode.
Under ideal conditions, the other ways should not reach the threshold so that their gain would always be less than the winning of the main mode. Practically, the difference is very small and one or two neighbouring modes, on each side of the main mode, carry an important part of laser power along with the main mode.
Such a laser is called a multimode semiconductor laser. Since each mode propagates inside the fiber at slightly different speeds due to group dispersion, the multimode nature of the lasers limits the distance-to-speed product to less than 10 (Gb /s) *km for systems running at 1.55 microns.

Laser Characteristics

- Continuous wave characteristics
- Modulation characteristics, small signal
- Modulation characteristics, high signal
- Laser noise



Laser characteristics in continuous wave

$$\begin{cases} \dfrac{dP}{dt} = GP + R_{sp} - \dfrac{P}{\tau_p} & (88) \\ \dfrac{dN}{dt} = \dfrac{I}{q} - \dfrac{N}{\tau_c} - GP & (89) \end{cases}$$

$$G = \Gamma \mathrm{v}_g g_m = G_N \left(N - N_0 \right) \quad (90$$

$$G_N = \Gamma \mathrm{v}_g \, \sigma_g / V , \quad N_0 = N_T V$$

$$\tau_p^{-1} = \mathrm{v}_g \alpha_{cav} = \mathrm{v}_g \left(\alpha_{mir} + \alpha_{\mathrm{int}} \right) \quad (91)$$

$$I_{th}\left(T\right) = I_0 \exp\left(T / T_0\right) \quad (92)$$

Dynamic equations can be written heuristically considering various physical phenomena by which a number, P, of photons, and a number, N, of electrons, change over time within the active region.

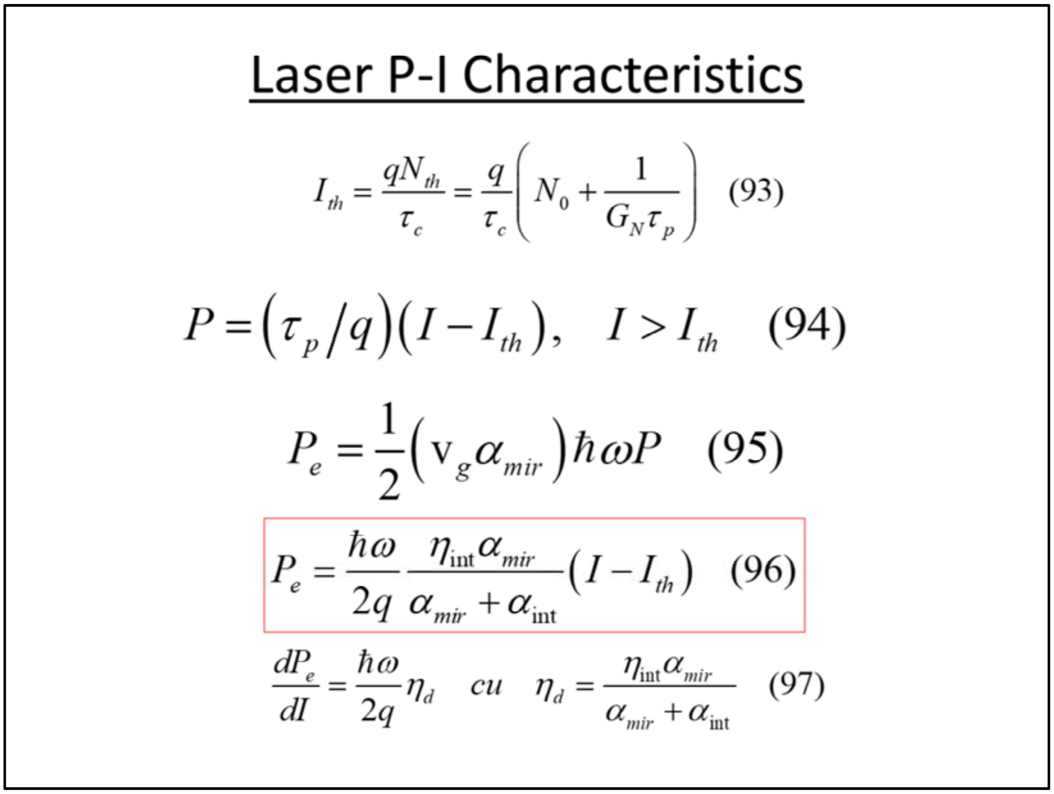
For a mono-mode laser, these equations are (88) - (89). G is the stimulated emission rate and R_sp is the spontaneous emission rate in laser mode. Note that R_sp is much smaller than the total rate in spontaneous emission - R_spon (ω). The reason is that spontaneous emission occurs in all directions in a broad spectrum (~ 30 - 40 nm), but only a small fraction of it, propagating along the cavity axis and emitted at laser frequency, actually contributes to eq. (88). In fact, R_sp and G are connected by the relation R_sp = n_sp * G, where n_sp is the spontaneous emission factor, having a value of about 2 for a semiconductor laser.

In equation (90), v_g is group speed, gamma is the confinement factor, and g_m is the gain of the material at the frequency of the mode. The last term in relation (88) takes into account the loss of photons in the cavity. The parameter τ_p is referred to as photon lifetime. This lifetime is related to the cavity losses, α_cav, through the relationship (91).

The three terms in the relationship (89) indicate the speeds with which the electrons are created or destroyed in the active region.

Curve P-I describes the properties of the semiconductor laser emission. The curves in the figure are for a laser on InGaAsP, at 1.3 microns and temperatures between 10 - 130 C. At room temperature, the threshold is reached at approx. 20 mA, the laser can emit 10 mW, through each face, at an applied current of 100 mA.

Laser performance degrades with increasing temperature. The threshold current increases exponentially with the temperature, rel (92), where I_0 is a constant and T_0 is a characteristic temperature. For lasers in INGaAsP, T_0 is typically between 50-70 K. Due to the temperature sensitivity, it is often necessary to control the temperature with a thermoelectric radiator.



Laser P-I Characteristics

$$I_{th} = \frac{qN_{th}}{\tau_c} = \frac{q}{\tau_c}\left(N_0 + \frac{1}{G_N \tau_p}\right) \quad (93)$$

$$P = \left(\tau_p / q\right)\left(I - I_{th}\right), \quad I > I_{th} \quad (94)$$

$$P_e = \frac{1}{2}\left(\mathrm{v}_g \alpha_{mir}\right)\hbar\omega P \quad (95)$$

$$\boxed{P_e = \frac{\hbar\omega}{2q}\frac{\eta_{\text{int}}\alpha_{mir}}{\alpha_{mir} + \alpha_{\text{int}}}\left(I - I_{th}\right) \quad (96)}$$

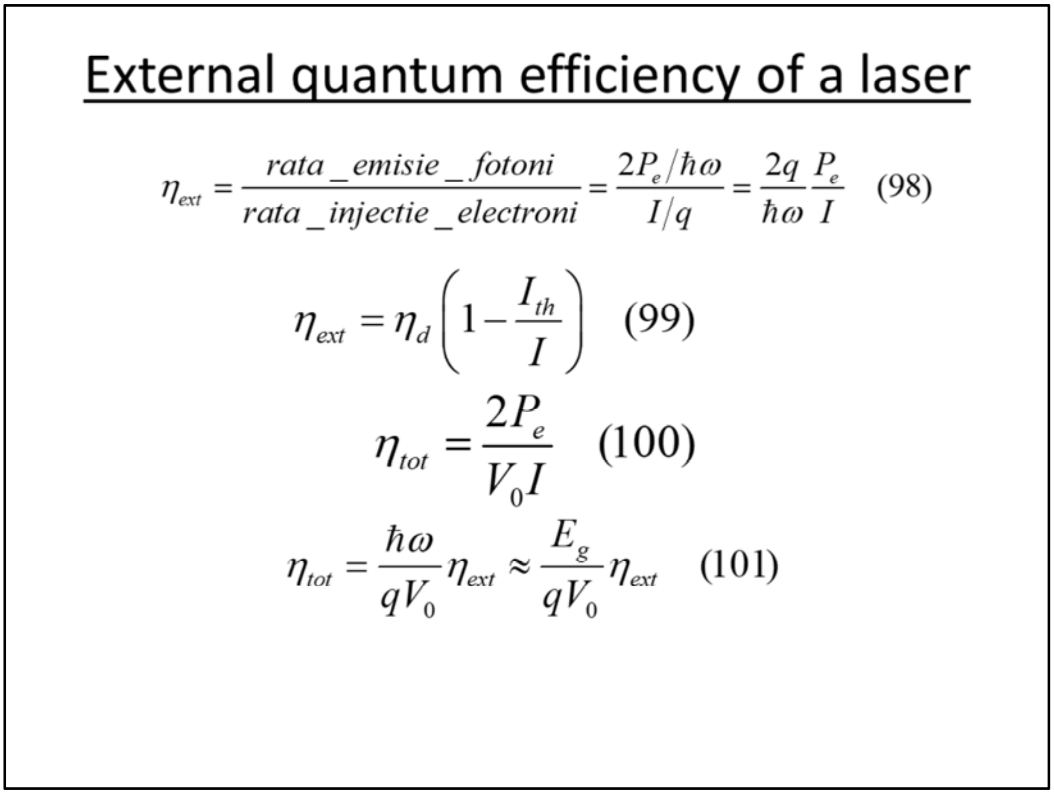
$$\frac{dP_e}{dI} = \frac{\hbar\omega}{2q}\eta_d \quad cu \quad \eta_d = \frac{\eta_{\text{int}}\alpha_{mir}}{\alpha_{mir} + \alpha_{\text{int}}} \quad (97)$$

The solution of the velocity equations can be simplified if spontaneous emission is neglected, R_sp = 0. For currents for which G * τ_p <1, P = 0 and N = τ_c * I / q. The threshold is reached for the current at which G * τ_p = 1. The carrier population is then limited to the threshold value N_th = N_0 + (G_N * τ_p) ^ (- 1). The threshold current will be given by the relationship (93).

For I> Ith, the number of photons P increases linearly with I, rel. (94).

The power emitted P_e is related to P by the relationship (95). In relation (95), v_g * α_mir is the speed at which the photon energy ħ * ω escapes from the cleaved faces. Factor ½ makes P_e the power emitted by each face of the FP cavity at equal reflections. Using equation (91_slide 35) and equation (94) in equation (95), we obtain the equation of emitted power (96), where n_int is introduced phenomenologically to indicate the fraction of injected electrons that is converted into photons by stimulated emission. In over-threshold mode, η_int is almost 100% for most semiconductor lasers.

A magnitude of practical interest is the slope of the P-I characteristic for I> Ith. This is called laser efficiency and is defined by the relationship (97). η_d is called *the differential quantum efficiency*



External quantum efficiency of a laser

$$\eta_{ext} = \frac{rata\_emisie\_fotoni}{rata\_injectie\_electroni} = \frac{2P_e/\hbar\omega}{I/q} = \frac{2q}{\hbar\omega}\frac{P_e}{I} \quad (98)$$

$$\eta_{ext} = \eta_d\left(1 - \frac{I_{th}}{I}\right) \quad (99)$$

$$\eta_{tot} = \frac{2P_e}{V_0 I} \quad (100)$$

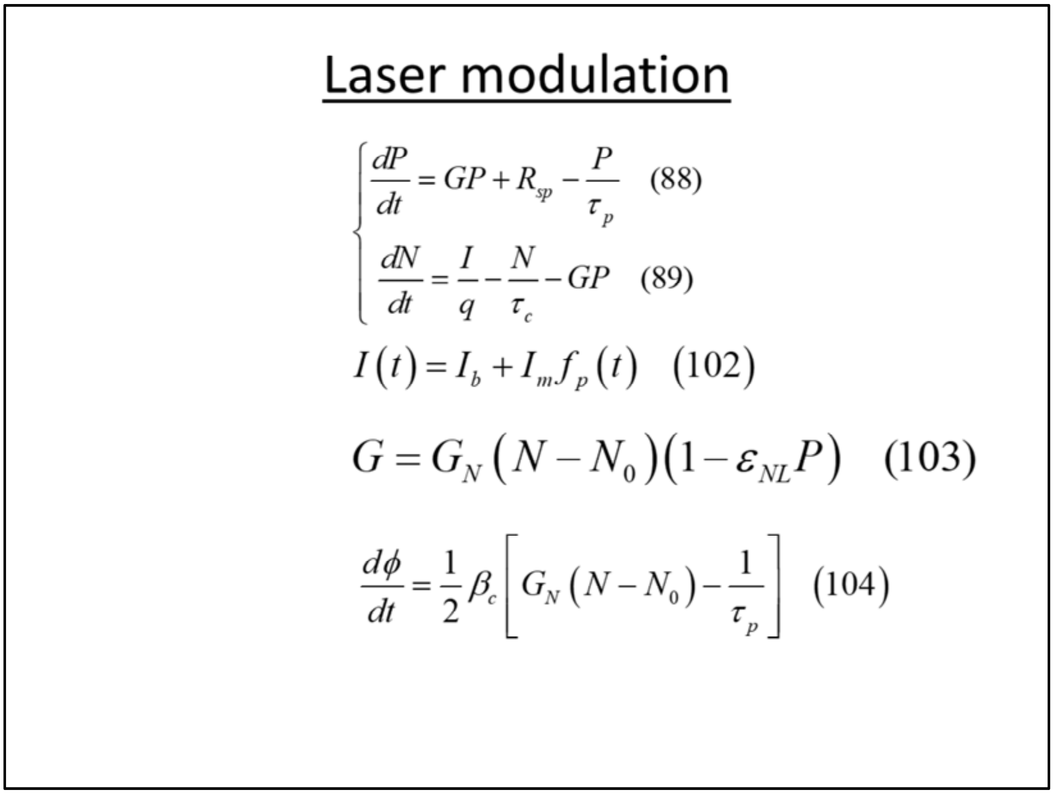
$$\eta_{tot} = \frac{\hbar\omega}{qV_0}\eta_{ext} \approx \frac{E_g}{qV_0}\eta_{ext} \quad (101)$$

We can define the external quantum efficiency through the relationship (98)

Relationship (99) express the link between η_ext and η_d. In general, the η_ext < η_d, but they become almost equal for I >> Ith.

Similar to LED, we can define total quantum efficiency as the relation (100), where V_0 is the voltage applied to the laser. Total quantum efficiency relates to external quantum efficiency through relationship (101).

In general, the_η_tot <η_ext as the applied voltage exceeds E_g / q. For lasers on GaAs, η_d can exceed 80% and η_tot can be approx. 50%. For Lasers on InGaAsP, η_d ~ 50% and η_tot ~ 20%.



Laser modulation

$$\begin{cases} \dfrac{dP}{dt} = GP + R_{sp} - \dfrac{P}{\tau_p} & (88) \\ \dfrac{dN}{dt} = \dfrac{I}{q} - \dfrac{N}{\tau_c} - GP & (89) \end{cases}$$

$$I(t) = I_b + I_m f_p(t) \quad (102)$$

$$G = G_N \left(N - N_0\right)\left(1 - \varepsilon_{NL} P\right) \quad (103)$$

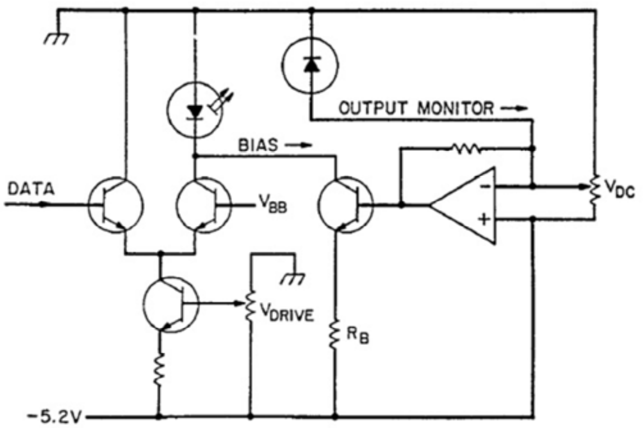
$$\frac{d\phi}{dt} = \frac{1}{2}\beta_c \left[G_N \left(N - N_0\right) - \frac{1}{\tau_p} \right] \quad (104)$$

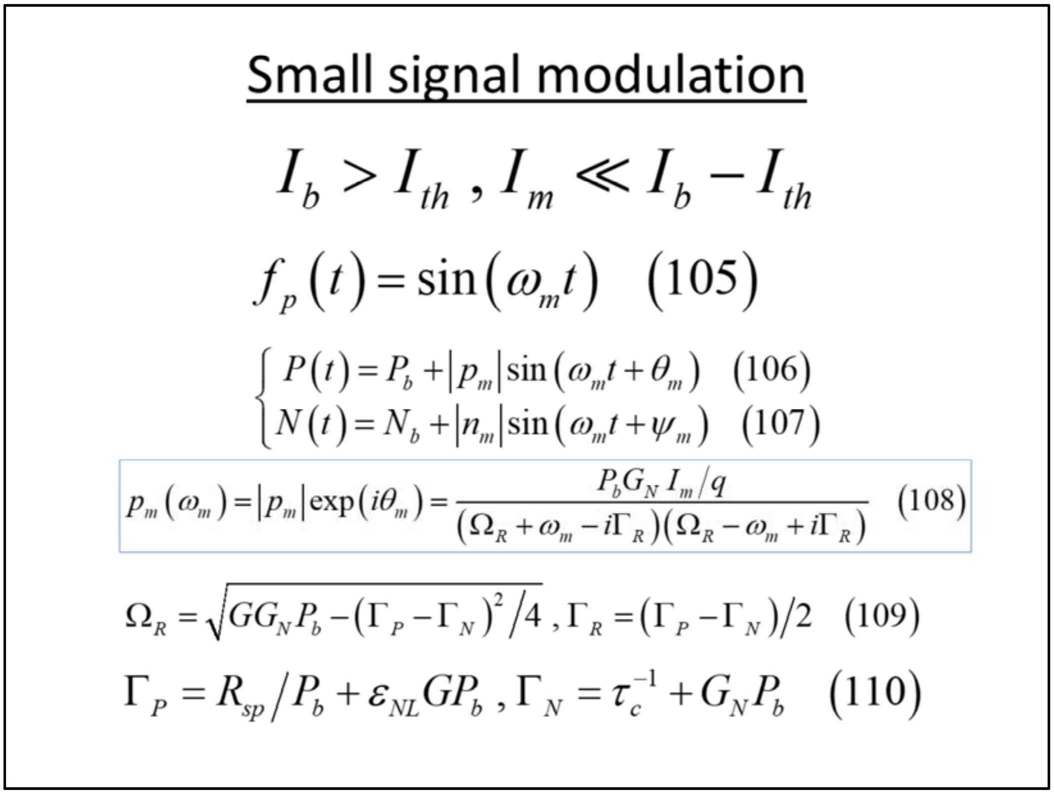
The modulation response of the semiconductor laser can be studied using the equations (88) and (89) where the current has the form of the relation (102). Ib is the polarization current, Im is the modulation current and fp (t) is the form of the pulse current.
Two changes must be added to these equations.

The first is to change the expression of the gain G in the form of the relation (103), where ε_NL is a nonlinearity parameter which leads to a slight reduction of the gain with the increase in the output power P. The typical value of this parameter is ε_LN ¬ 10 ^ (- 7) . The relation (90) of slide 37 is valid for ε_NL * P << 1.

The second change is related to the fact that whenever the gain is changed as a result of the modification of the populations of electrons, N, the refractive index of the semiconductor changes. This makes the amplitude modulation in the semiconductor lasers always accompanied by a phase modulation. This phase modulation can be considered by including equation (104), wherein β_c is *the amplitude-phase coupling parameter*.

Laser modulation - 2





Small signal modulation

$$I_b > I_{th}\ ,\ I_m \ll I_b - I_{th}$$

$$f_p(t) = \sin(\omega_m t) \quad (105)$$

$$\begin{cases} P(t) = P_b + |p_m|\sin(\omega_m t + \theta_m) & (106) \\ N(t) = N_b + |n_m|\sin(\omega_m t + \psi_m) & (107) \end{cases}$$

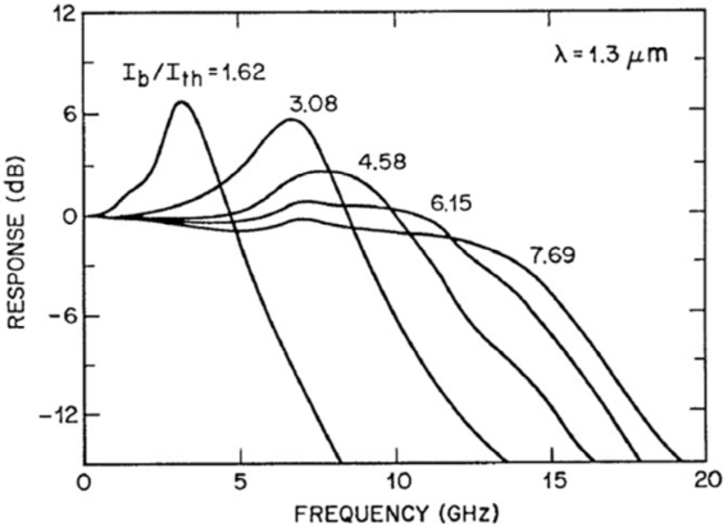
$$p_m(\omega_m) = |p_m|\exp(i\theta_m) = \frac{P_b G_N I_m / q}{(\Omega_R + \omega_m - i\Gamma_R)(\Omega_R - \omega_m + i\Gamma_R)} \quad (108)$$

$$\Omega_R = \sqrt{G G_N P_b - (\Gamma_P - \Gamma_N)^2/4}\ ,\ \Gamma_R = (\Gamma_P - \Gamma_N)/2 \quad (109)$$

$$\Gamma_P = R_{sp}/P_b + \varepsilon_{NL} G P_b\ ,\ \Gamma_N = \tau_c^{-1} + G_N P_b \quad (110)$$

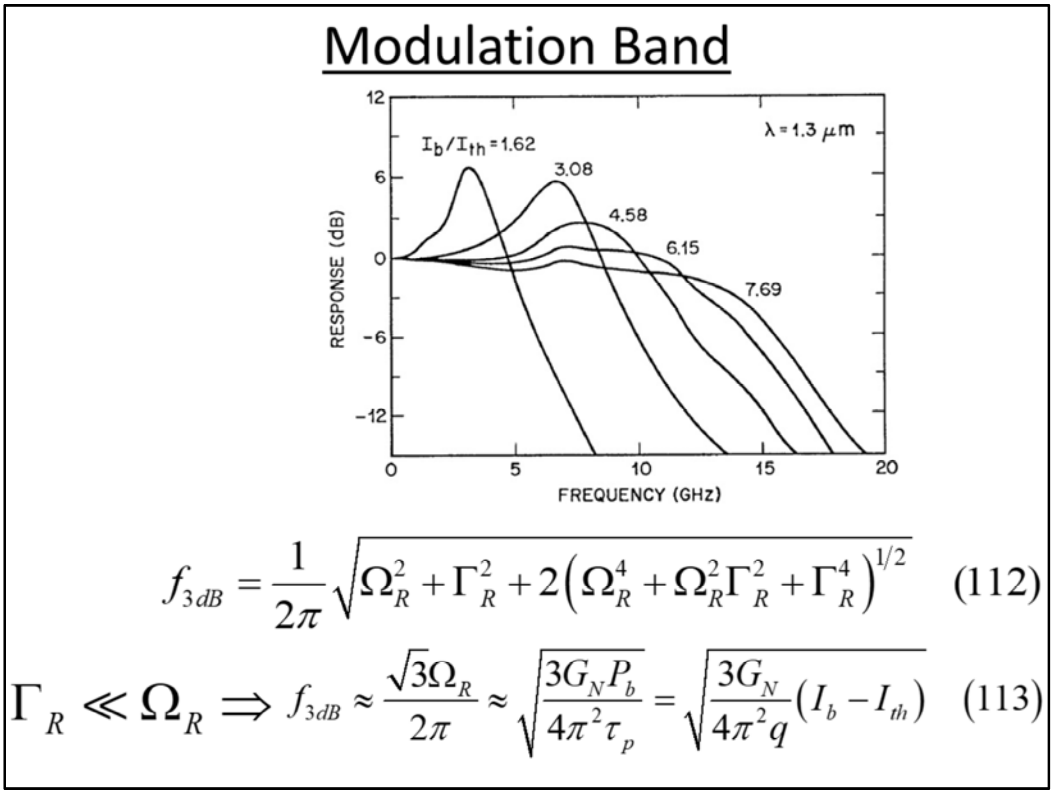
In case of low signal conditions, the dynamic equations can be linearized and analyzed analytically using the Fourier transform for fp (t). The low signal modulation band is obtained by considering the laser response to sinusoidal modulation, for which f_p is given in relation (105). The solutions of the dynamic equations are the relations (106) - (107), where Pb and Nb are stationary values at the polarization current Ib, pm and nm are small changes due to the current modulation, and θ_m and ψ_m are the phases associated with low signal modulation . In particular we have the relation (108), where Ω_R and Γ_R are frequent and the attenuation velocity of the damped oscillations given by the relationships (109) and respectively (110). The modulation efficiency decreases when the modulation frequency exceeds Ω_R with a high value.

Tranfer Function



$$H\left(\omega_m\right)=\frac{p_m\left(\omega_m\right)}{p_m(0)}=\frac{\Omega_R^2+\Gamma_R^2}{\left(\Omega_R+\omega_m-i\Gamma_R\right)\left(\Omega_R-\omega_m+i\Gamma_R\right)} \quad (111)$$

The modulation response is flat (H (ω_m) ¬1) for frequencies ω_m << Ω_R, the maximum value is reached for equality and then there is a sudden drop for ω_m >> Ω_R.



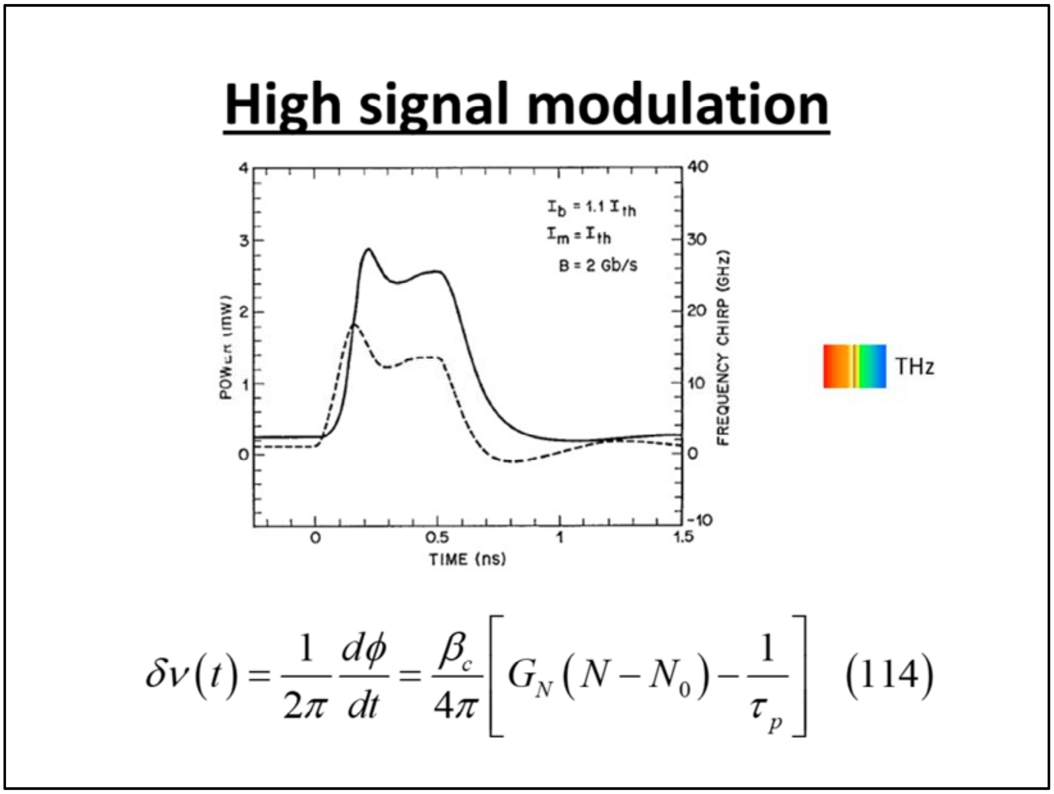
Modulation Band

$$f_{3dB} = \frac{1}{2\pi}\sqrt{\Omega_R^2 + \Gamma_R^2 + 2\left(\Omega_R^4 + \Omega_R^2\Gamma_R^2 + \Gamma_R^4\right)^{1/2}} \tag{112}$$

$$\Gamma_R \ll \Omega_R \Rightarrow f_{3dB} \approx \frac{\sqrt{3}\Omega_R}{2\pi} \approx \sqrt{\frac{3G_N P_b}{4\pi^2 \tau_p}} = \sqrt{\frac{3G_N}{4\pi^2 q}\left(I_b - I_{th}\right)} \tag{113}$$

The modulation band at 3-dB is defined as the modulation frequency at which the (H (ω_m)) mode is reduced by 3 dB (i.e., 2 times) compared to the continuous current value.

The relationship (113) shows that the modulation band increases with increasing the polarization level, proportional to sqr (Pb) or sqr ((Ib-Ith)).



High signal modulation

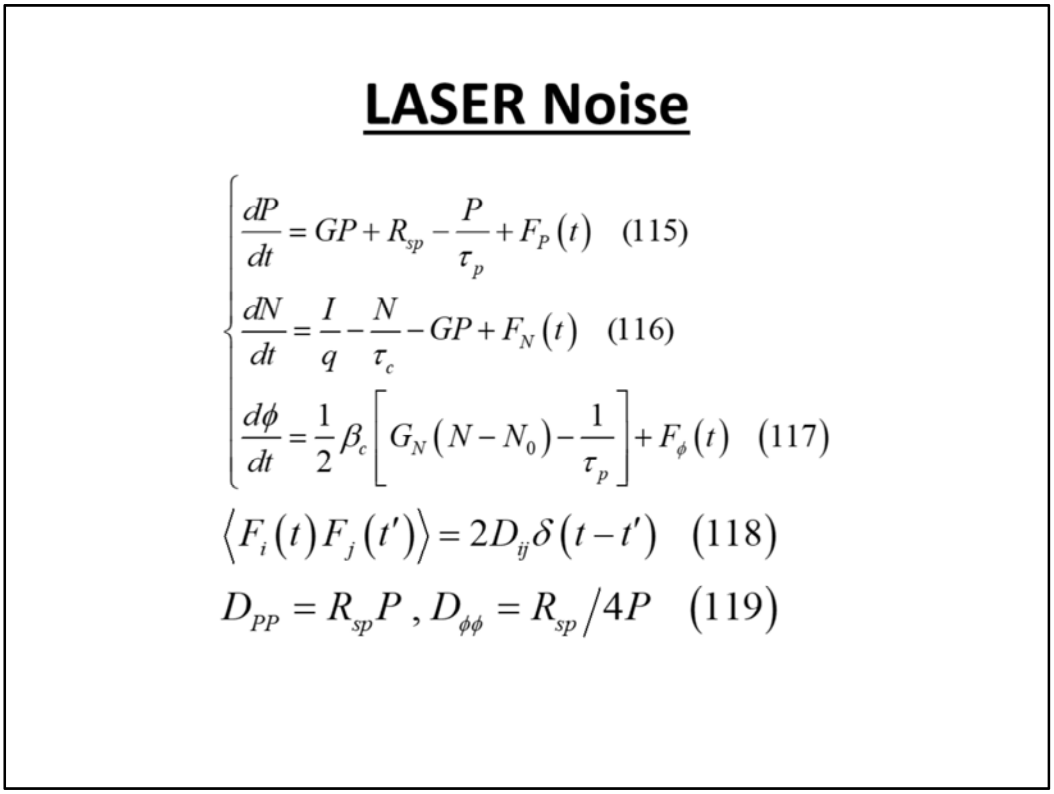
$$\delta\nu(t) = \frac{1}{2\pi}\frac{d\phi}{dt} = \frac{\beta_c}{4\pi}\left[G_N\left(N - N_0\right) - \frac{1}{\tau_p}\right] \quad (114)$$

Small signal analysis does not apply to the practical use of laser diodes, in which the diode is polarized near the threshold and modulated well above the threshold to obtain light pulses representing the bits to be transmitted.

In this case, the dynamic equations must be solved numerically. The figure shows an example for a polarized laser with Ib = 1.1 * Ith and modulated at 2 Gb / s using a current pulse duration of 500 ps and amplitude Im = Ith. Optical pulse has a growth time of ¬ 100 ps and a drop of ¬300 ps.

As mentioned above, the amplitude modulation in a semiconductor laser is accompanied by a phase modulation. A time-varying phase is equivalent to transient changes in the mode frequency from stationary value ν_0. Such a pulse is called 'chirped'. The chirp frequency, Sv (t), is obtained from the dynamic phase equation and has the expression (114).

In the figure, the dotted curve shows the chirp frequency in the optical pulse. The frequency of the mode moves towards the blue around the front front and towards the red front side of the optical pulse. This shift means that the pulse spectrum is considerably wider than expected in the absence of the phenomenon.



LASER Noise

$$\begin{cases} \dfrac{dP}{dt} = GP + R_{sp} - \dfrac{P}{\tau_p} + F_P(t) \quad (115) \\ \dfrac{dN}{dt} = \dfrac{I}{q} - \dfrac{N}{\tau_c} - GP + F_N(t) \quad (116) \\ \dfrac{d\phi}{dt} = \dfrac{1}{2}\beta_c\left[G_N(N - N_0) - \dfrac{1}{\tau_p}\right] + F_\phi(t) \quad (117) \end{cases}$$

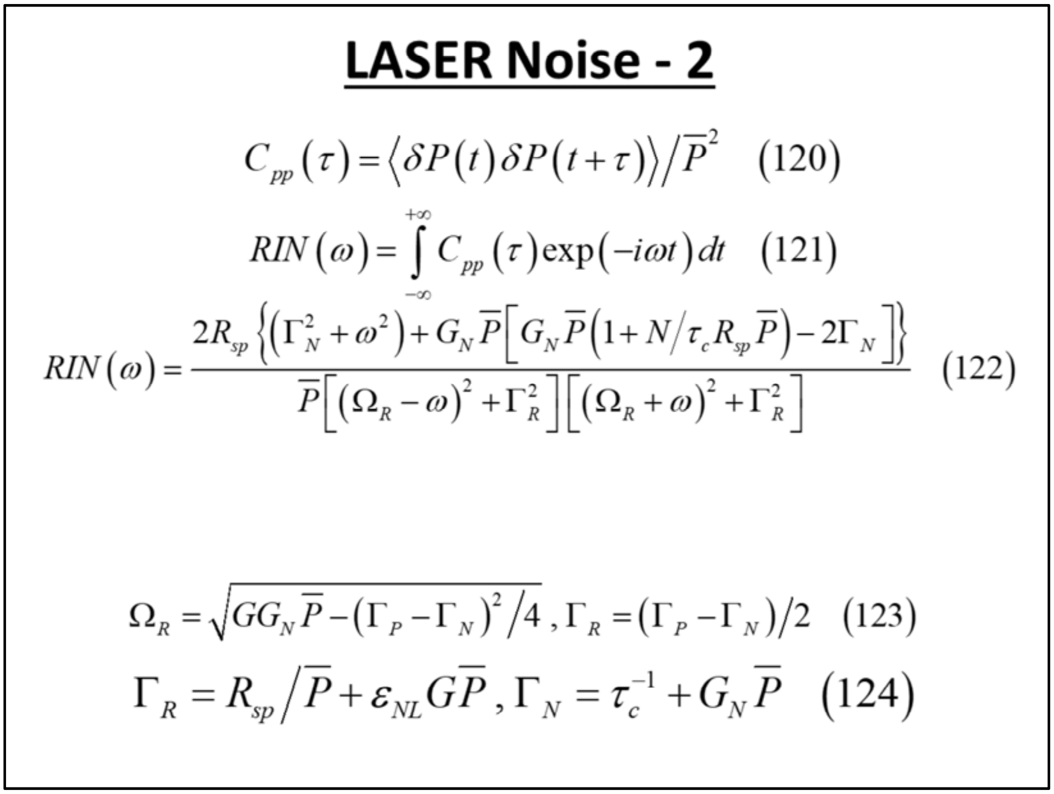
$$\langle F_i(t) F_j(t') \rangle = 2D_{ij}\delta(t - t') \quad (118)$$

$$D_{PP} = R_{sp}P \text{ , } D_{\phi\phi} = R_{sp}/4P \quad (119)$$

The output of a semiconductor laser shows fluctuations in intensity, phase and frequency even when the laser is polarized at a constant current with negligible current fluctuations.

The fundamental mechanisms of noise are spontaneous emission and electron-empty recombination (alice noise). Noise in semiconductor lasers is dominated by spontaneous emission. Each spontaneously emitted photon adds to the coherent field a small field component whose phase is random and thus disrupts both the amplitude and phase, randomly. Moreover, such spontaneous emission events occur randomly at a speed of 10-12 per second because the semiconductor lasers have a relatively high value of R_sp. The result is that the intensity and the phase of the light emitted are fluctuations in an orderly 100-second time scale. These intensity fluctuations limit the signal-to-noise ratio (SNR), and phase fluctuations lead to a finite line width when the laser is fed to the constant current. Because these fluctuations affect the performance of optical systems, it is important to evaluate their size.

Dynamics equations can be used to study laser noise by adding a noise term known as the Langevin force, equations (115) - (117). These are considered Gaussian processes with zero mean and having the correlation function of form (118) (Markov approximation). i, j = P, N, Φ, and D_ij are the diffusion coefficients given in the relations (119).



LASER Noise - 2

$$C_{pp}(\tau) = \langle \delta P(t)\, \delta P(t+\tau) \rangle / \overline{P}^2 \quad (120)$$

$$RIN(\omega) = \int_{-\infty}^{+\infty} C_{pp}(\tau) \exp(-i\omega t)\, dt \quad (121)$$

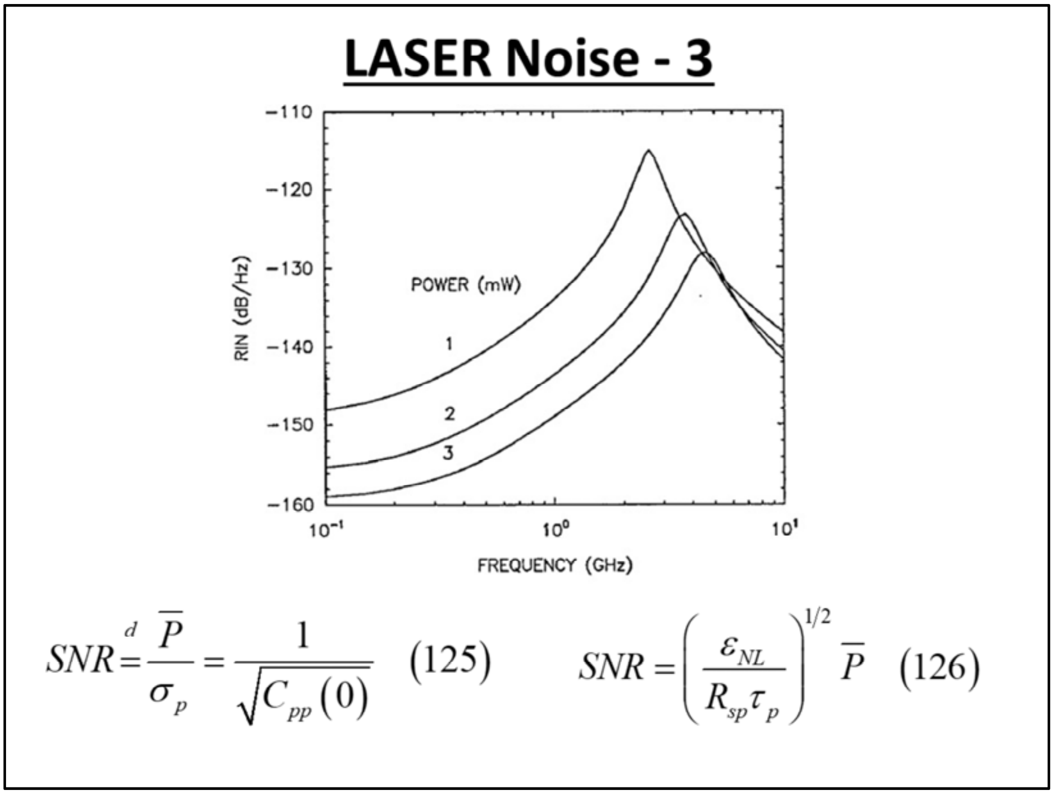
$$RIN(\omega) = \frac{2R_{sp}\left\{\left(\Gamma_N^2 + \omega^2\right) + G_N \overline{P}\left[G_N \overline{P}\left(1 + N/\tau_c R_{sp} \overline{P}\right) - 2\Gamma_N\right]\right\}}{\overline{P}\left[\left(\Omega_R - \omega\right)^2 + \Gamma_R^2\right]\left[\left(\Omega_R + \omega\right)^2 + \Gamma_R^2\right]} \quad (122)$$

$$\Omega_R = \sqrt{G G_N \overline{P} - \left(\Gamma_P - \Gamma_N\right)^2 / 4}\,, \Gamma_R = \left(\Gamma_P - \Gamma_N\right)/2 \quad (123)$$

$$\Gamma_R = R_{sp} / \overline{P} + \varepsilon_{NL} G \overline{P}\,, \Gamma_N = \tau_c^{-1} + G_N \overline{P} \quad (124)$$

The function of autocorrelation of the light intensity is defined by the relation (120), where δP (t) represents the small fluctuation. The Fourier transform of the autocorrelation function is the RIN (Relative Intensity Noise), the relationship (121). The RIN can be calculated by linearizing the relations (1) and (2) from the previous slide in delta_P and delta_N, using the linearization in the frequency domain and calculating the mean using the relationship (4) of the previous slide.

We obtain the relationship (122), where Ω_R and Γ_R are frequent and the rate of attenuation of the amortized oscillations, where P_b has been replaced by <P>, the relations (123), (124).



LASER Noise - 3

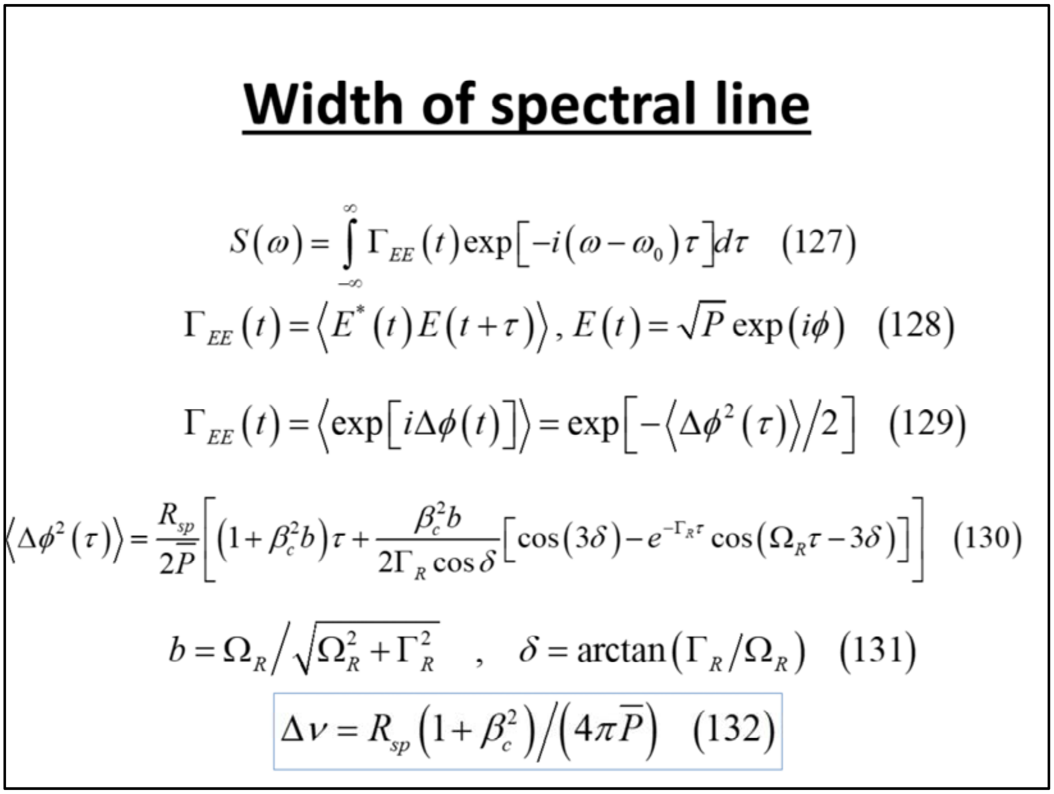
$$SNR \overset{d}{=} \frac{\overline{P}}{\sigma_p} = \frac{1}{\sqrt{C_{pp}(0)}} \quad (125)$$

$$SNR = \left(\frac{\varepsilon_{NL}}{R_{sp}\tau_p}\right)^{1/2} \overline{P} \quad (126)$$

The figure shows RIN calculated at different power levels for a laser diode on InGaAsP at 1.55 microns. RIN increases considerably beyond the relaxation oscillations Ω_R and decreases rapidly for >> Ω_R.

In essence, the laser acts as a conductor passing the band, having the band Ω_R, on spontaneous emission fluctuations. At a given frequency, RIN decreases with increasing laser power by a law P ^ (-3).
For small powers, behavior changes according to a law P ^ (-1).

The magnitude of practical interest is SNR, defined as the relationship (125). At power levels over several milliwatts, the SNR exceeds 20 dB and increases according to the law of the relationship (126)



Width of spectral line

$$S(\omega)=\int_{-\infty}^{\infty}\Gamma_{EE}(t)\exp\left[-i(\omega-\omega_0)\tau\right]d\tau \quad (127)$$

$$\Gamma_{EE}(t)=\left\langle E^*(t)E(t+\tau)\right\rangle,\ E(t)=\sqrt{P}\exp(i\phi) \quad (128)$$

$$\Gamma_{EE}(t)=\left\langle \exp\left[i\Delta\phi(t)\right]\right\rangle=\exp\left[-\left\langle\Delta\phi^2(\tau)\right\rangle/2\right] \quad (129)$$

$$\left\langle\Delta\phi^2(\tau)\right\rangle=\frac{R_{sp}}{2\bar{P}}\left[\left(1+\beta_c^2 b\right)\tau+\frac{\beta_c^2 b}{2\Gamma_R\cos\delta}\left[\cos(3\delta)-e^{-\Gamma_R\tau}\cos\left(\Omega_R\tau-3\delta\right)\right]\right] \quad (130)$$

$$b=\Omega_R\Big/\sqrt{\Omega_R^2+\Gamma_R^2}\quad,\quad \delta=\arctan\left(\Gamma_R/\Omega_R\right) \quad (131)$$

$$\boxed{\Delta\nu=R_{sp}\left(1+\beta_c^2\right)\Big/\left(4\pi\bar{P}\right) \quad (132)}$$

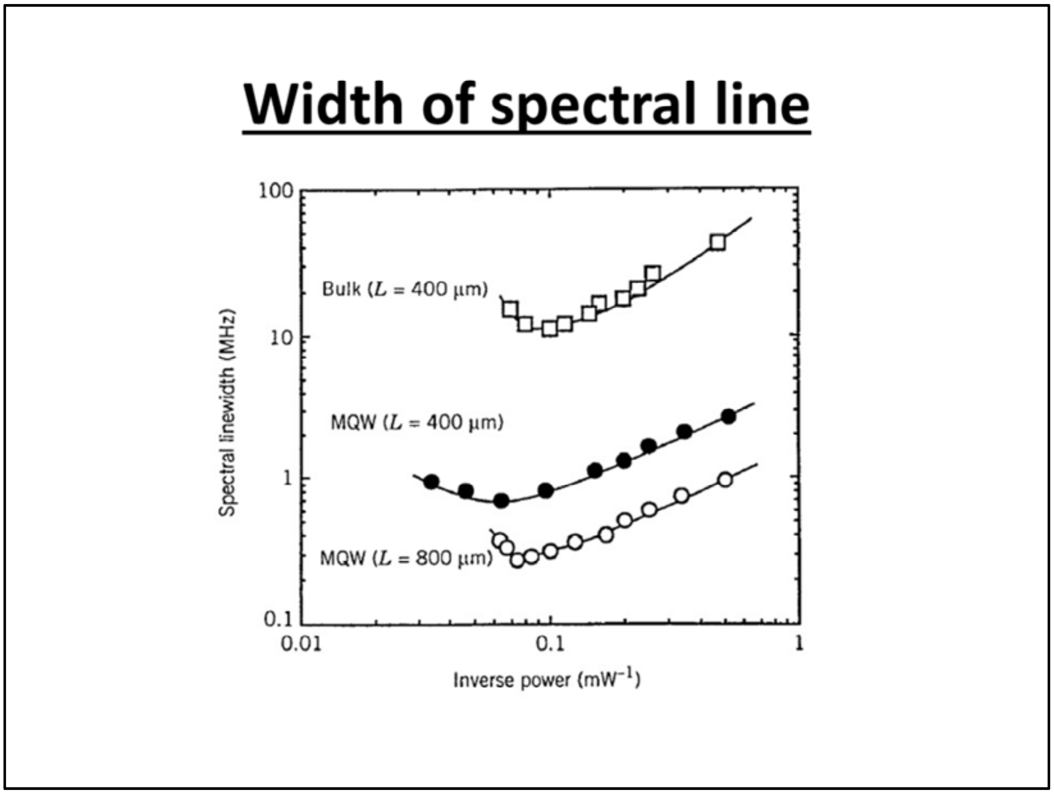
The spectrum of light emitted is related to the autocorrelation function of the field, C_EE, by Fourier transform, the relation (126). In relation (128) the autocorrelation function of the optical field is defined. If the intensity fluctuations are neglected, C_EE can be written as (129), where ΔΦ is considered a Gaussian process. The phase variant can be calculated by linearizing the dynamic equations, the result being given in relation (130), where b and δ are given in relations (131).

The spectrum is dominated by a maximum located at the frequency ω_0 and accompanied by many satellites at the frequencies ω_0 +/- m * Ω_R, where m is integer. The amplitudes of the satellites are at most 1% of the amplitude of the main peak.

These satellites are given by relaxation oscillations, which are responsible for the term b.

If these oscillations are neglected, b = 1 and the autocorrelation function of the field decreases exponentially with τ. In this case, the integral (127) can be calculated analytically and the spectrum is found to be Lorentzian.

The spectral width, Δv, of this Lorentzian spectrum, defined at half of the maximum (FWHM), is given by the relationship (132), where C_R << Ω_R is assumed. The spectral width is increased by a factor equal to (1 + β 2) as a result of the amplitude-phase coupling governed by parameter b; this is why β is called the spectral width increase factor.



Width of spectral line

Δv varies with L ^ (- 2), which included dependence of Rsp and P.
L is the length of the laser cavity.